Pre-Lab: Primer on Experimental Errors
There are no points assigned for this Pre-Lab.

An essential skill in the repertoire of an experimental physicist is his/her ability to identify the types of errors present in an experiment, to assess their size, and to manage them effectively. In the language of experimental physics, an error is not a mistake; rather it is an inherent limitation or uncertainty in any measurement or set of measurements. Errors arise from imperfect people making imperfect measurements using imperfect equipment in an imperfect laboratory. Needless to say, errors are always present; however their influence can be controlled and, hopefully, minimized.

Types of Errors
Errors may be classified as either systematic or random.

Systematic errors have causes that are known or can be determined. These errors can arise from slight flaws in the construction or the calibration of instruments which tend to shift measurements in a specific direction (e.g. too low or too high). The inability to read precisely the final digit of a measurement made with an instrument that has an “analogue” scale (such as a meterstick or a dial clock) is another example of systematic error. Systematic errors can be controlled and, in some cases, completely eliminated.

In PHYS 152/251 Laboratory, we assume that the systematic error equals the smallest scale division. For example, the smallest division on a standard meterstick is 1 mm, so the systematic error in measuring length \( L \) using a meterstick is 1 mm, or \( \delta L = 1 \) mm. We use the Greek lower-case delta \( \delta \) to mean “the error in.”

Random errors have causes that are unknown or cannot be determined. These errors cannot be completely controlled nor entirely eliminated. They are indeed random but their effect tends to decrease whenever repeated measurements of the same quantity are taken. Statistics are used to describe data sets with random error.

Suppose we take \( N \) measurements of physical quantity \( x: x_1, x_2, \ldots, x_N \). The mean (or average) value of the set

\[
\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i
\]

is its characteristic (or “best”) value for \( x \). The (sample) standard deviation
\[ \sigma_x = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2} \]
gives a measure of the random error. Note the factor of \( N - 1 \) in the denominator. As \( N \) increases, \( \sigma_x \) decreases and the data tends to cluster around \( \bar{x} \) in a so-called normal (or bell-shaped) distribution. The “true” value of \( x \) has a 68.3\% probability of lying between \( \bar{x} - \sigma_x \) and \( \bar{x} + \sigma_x \).

Suppose you measure the period \( T \) of a simple pendulum ten times using a stopwatch: 1.99 s, 1.98 s, 2.01 s, 2.00 s, 1.98 s, 1.99 s, 1.97 s, 2.01 s, 2.00 s, 1.99 s.
Using a TI-30Xa calculator, we find a mean value \( \bar{T} = 1.99 \) s with a standard deviation \( \sigma_T = 0.01 \) s. The true value of \( T \) has a 68.3\% probability of lying between 1.98 s and 2.00 s.

**Error Propagation**

Usually it is not possible to make a direct measurement of the certain physical quantities, such as the acceleration of gravity \( g \). We must instead measure secondary quantities (e.g. distance and time) and then use those measurements in a formula to calculate the quantity of primary interest.

In a free fall experiment, the acceleration of gravity may be calculated from the formula
\[ g = \frac{2h}{t^2}, \text{ where } h \text{ is the height at which an object is dropped and } t \text{ is the time for its fall.} \]

Given that our height measurement carries an error \( \delta h \) and our time measurement carries an error \( \delta t \), it is necessary not only to calculate \( g \) but also its “propagated” error \( \delta g \).

The rules for error propagation are derived from elementary calculus. Only the results are stated below. In each rule, \( Q \) represents the derived quantity of propagated error \( \delta Q \), which depends on measured variables \( A, B, C, \ldots \) with errors (systematic or random) \( \delta A \), \( \delta B \), \( \delta C \), \ldots, respectively. It is often convenient to express errors in *fractional* (or *relative*) form: \( \frac{\delta Q}{Q}, \frac{\delta A}{A} \), etc.

I. If \( Q = kA \), where \( k \) is a mathematical constant, \( \delta Q = k \delta A \) or \( \frac{\delta Q}{Q} = \frac{\delta A}{A} \).
II. If \( Q = kA^m \), where \( k \) is a mathematical constant and \( m \) is any power,
\[
\delta Q = k m A^{m-1} \delta A \quad \text{or} \quad \frac{\delta Q}{Q} = m \cdot \frac{\delta A}{A}
\]

III. If \( Q = A + B \) or \( Q = A - B \), \( \delta Q = \sqrt{(\delta A)^2 + (\delta B)^2} \)

IV. If \( Q = k A^m B^n \), where \( k \) is a mathematical constant,
\[
\frac{\delta Q}{Q} = \sqrt{\left(m \cdot \frac{\delta A}{A}\right)^2 + \left(n \cdot \frac{\delta B}{B}\right)^2}
\]

V. If \( Q = f(A, B, C, \ldots) \) in general, \( \delta Q = \sqrt{\left(\frac{\partial f}{\partial A} \cdot \delta A\right)^2 + \left(\frac{\partial f}{\partial B} \cdot \delta B\right)^2 + \left(\frac{\partial f}{\partial C} \cdot \delta C\right)^2 + \ldots} \)

where \( \frac{\partial f}{\partial A} \) is the partial derivative of \( f \) with respect to \( A \), etc.

In the example above, the drop height is measured to be 2.450 m with a systematic error of 0.001 m. We time ten drops, giving a mean of 0.73 s with a standard deviation of 0.02 s. The formula for the acceleration of gravity is \( \frac{2h}{t^2} \), so \( g = \frac{2(2.450 \text{ m})/(0.73 \text{ s})^2}{2} = 9.2 \text{ m/s}^2 \). (Result must have 2 significant figures because 0.73 s has only 2 sig figs.)

The equation for \( g \) conforms to Rule IV (i.e. \( g = 2 \cdot h \cdot t^{-2} \)), so
\[
\frac{\delta g}{g} = \sqrt{\left(\frac{\delta h}{h}\right)^2 + \left(-2 \cdot \frac{\delta t}{t}\right)^2} = \sqrt{\left(\frac{0.001}{2.450}\right)^2 + \left(-2 \cdot \frac{0.02}{0.73}\right)^2} = 0.055, \text{ and}
\]
\[
\delta g = 0.055g = 0.055(9.2 \text{ m/s}^2) = 0.5 \text{ m/s}^2 \text{ (Last digit must have same decimal place as } g)\)

We record our final result in the format \( g \pm \delta g = 9.2 \pm 0.5 \text{ m/s}^2 \).

Graphing

Physicists often graph their data in order to better detect mathematical relationships between measured quantities. The simplest relationship between quantities \( x \) and \( y \) is a straight line: \( y = Ax + B \), where \( A \) is the slope and \( B \) is the vertical intercept. Of course, we do not expect that all of our data points \((x, y)\) lie exactly along a straight line; however we anticipate that most data points will not “stray” too far from it. The line that “best fits” the data points may be calculated through the statistical method of least squares. Several experiments in this course will require that you find the best-fit line through your data and its associated error. You are not required to know how to do these calculations.
though you are expected to be familiar with the Chart features available on Microsoft Excel.

For example, the graph above is an “XY plot” obtained from Excel. The “Add Trendline” utility was used to graph the best-fit line. “Display equation on chart” and “Display R-squared value on chart” were selected.

The $R^2$ value is a measure of how well the data points cluster about the line. ($R$ itself is called the “correlation coefficient.”) $R^2$ is always between 0 and 1: $R^2 = 0$ indicates that the points are randomly scattered in the graph and therefore do not follow a straight line; $R^2 = 1$ indicates that all points lie exactly on the best-fit line. In this course, $R^2$ values will be used to determine the random errors in the slope $A$ and vertical intercept $B$ obtained from Excel. Specifically,

$$\frac{\delta A}{A} = 1 - R^2 \quad \text{and} \quad \frac{\delta B}{B} = 1 - R^2$$

From the graph above, $A = 0.2168$, $B = 1.8383$, and $R^2 = 0.9517$. So,

$$\frac{\delta A}{0.2168} = 1 - 0.9517 \Rightarrow \delta A = 0.0105 \quad \text{random error in slope } A$$

$$\frac{\delta B}{1.8383} = 1 - 0.9517 \Rightarrow \delta B = 0.0888 \quad \text{random error in vertical intercept } B$$
Agreement of Measured Values

One of the goals of experimentation in physics is to determine whether our final measured value for some physical quantity “agrees” with either (a) an accepted or standard value, or (b) a measured value obtained by an alternate procedure (i.e. using a different formula).

In situation (a) the accepted or standard value of our physical quantity is reported as a “useful constant” listed on the inside cover of the textbook, e.g. the acceleration of gravity is listed as 9.80665 m/s\(^2\) (standard). In the free fall experiment discussed above, we obtained \(g \pm \delta g = 9.2 \pm 0.5\) m/s\(^2\), which means that our final result lies between 8.7 m/s\(^2\) and 9.7 m/s\(^2\). Note that the standard value lies outside of this range, so we say that our measured value does not agree with the standard value. In general, if \(x \pm \delta x\) is our measured result and \(x_{\text{std}}\) is the standard textbook value, then \(x - \delta x < x_{\text{std}} < x + \delta x\) implies that the values agree; otherwise they disagree.

In situation (b), two different measurements of the same physical quantity, \(x_1 \pm \delta x_1\) and \(x_2 \pm \delta x_2\), are being compared. If these two ranges \([x_1 - \delta x_1, x_1 + \delta x_1]\) and \([x_2 - \delta x_2, x_2 + \delta x_2]\) overlap, then the values agree; otherwise they disagree. For example, suppose a different experiment measures the acceleration of gravity as 9.85 ± 0.20 m/s\(^2\), so our two ranges are [8.7 m/s\(^2\), 9.7 m/s\(^2\)] and [9.65 m/s\(^2\), 9.95 m/s\(^2\)], which indeed overlap so the measurements agree.