

Project 5.1a: Integration: Riemann Sums

Objective

To illustrate how Maple can be used to approximate an area using Riemann sums.

Narrative

If you have not already done so, read Section 5.1 of the text.

Finding the area under a curve using Riemann sums, while being conceptually straightforward, can be computationally challenging. In this project you will see how Maple can be used to simplify computations.

In this project we introduce the commands:

<code>sum(E(i), i=1..n)</code>	The numerical value of the sum of $E(i)$ from $i = 1$ to $i = n$.
<code>Sum(E(i), i=1..n)</code>	The symbol for the sum of $E(i)$ from $i = 1$ to $i = n$.
<code>int(f(x), x=a..b)</code>	The numerical value of the definite integral $\int_{x=a}^b f(x) dx$.
<code>Int(f(x), x=a..b)</code>	The symbol for the definite integral $\int_{x=a}^b f(x) dx$.

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at finding the area under the graph of $f(x) = x^2 + x$ from $x = 1$ to $x = 3$. The effect of each command is described in the right-hand column for your reference. Your lab report will be a hard copy of your typed input and Maple's responses (both text and graphics).

<code>> # Your name, today's date</code>	
<code>> # Project 5.1a: Integration: Riemann Sums</code>	
<code>> restart;</code>	Clear Maple's memory.
<code>> f := x -> x^2+x;</code>	Let $f(x) = x^2 + x$.
<code>> a := 1; b := 3;</code>	Let $a = 1$ and $b = 3$.
<code>> dx := (b-a)/n;</code>	Let $dx = (b - a)/n$.
<code>> LHSum := simplify(sum(f(a+(i-1)*dx)*dx, i=1..n));</code>	Let $LHSum$ denote the left-hand Riemann sum.
<code>> MPSum := simplify(sum(f(a+(2*i-1)*dx/2)*dx, i=1..n));</code>	Let $MPSum$ denote the midpoint Riemann sum.
<code>> RHSum := simplify(sum(f(a+i*dx)*dx, i=1..n));</code>	Let $RHSum$ denote the right-hand Riemann sum.

Observe that the x -coordinate x_i^* of the i th "sample point" $(x_i^*, f(x_i^*))$, $i = 1, \dots, n$, used to compute:

- $LHSum$ is $x_i^* = a + (i - 1)dx$,
- $RHSum$ is $x_i^* = a + i dx$, and
- $MPSum$ is $x_i^* = a + \frac{(i - 1) + i}{2}dx = a + \frac{2i - 1}{2}dx$

where x_i^* for $MPSum$ is just the average of the x_i^* 's for $LHSum$ and $RHSum$.

b) Continue by typing the command line in the left-hand column below into Maple.

<code>> plot(f(x), x=a..b, y=0..f(b)); %; %;</code>	Draw <i>three</i> copies of the graph of f .
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Later, after you have made a hard copy of your typed input and Maple's responses, you will be asked to draw the rectangles and plot the sample points $(x_i^*, f(x_i^*))$ used to compute $LHSum$, $MPSum$, and $RHSum$ for $n = 4$ on these graphs.

c) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands allow us to compute $LHSum$, $MPSum$, and $RHSum$ as n goes to ∞ .

> for n from 4 to 204 by 20 do	
print(n,evalf(LHSum),evalf(MPSum),evalf(RHSum))	
end do:	
> n := 'n';	Redefine n as a variable.
> evalf(limit(LHSum,n=infinity));	Compute the limit of $LHSum$ as n goes to ∞ .
> evalf(limit(MPSum,n=infinity));	Compute the limit of $MPSum$ as n goes to ∞ .
> evalf(limit(RHSum,n=infinity));	Compute the limit of $RHSum$ as n goes to ∞ .
> Int(f(x),x=a..b) = evalf(int(f(x),x=a..b));	Find $\int_{x=a}^b f(x) dx$.

At this time make a hard copy of your typed input and Maple's responses. Then, ...

d) on the graphs you produced in part (b), draw the rectangles, and plot and label the sample points $(x_i^*, f(x_i^*))$, used to compute $LHSum$, $MPSum$, and $RHSum$ for $n = 4$.

Comments

1. Over the interval $[1, 3]$, $f(x) = x^2 + x$ is increasing (can you see how you might verify this without graphing?); thus the right-hand Riemann sum is associated with *circumscribed* rectangles, and the left-hand Riemann sum is associated with *inscribed* rectangles. Over the interval $[-3, -1]$, on the other hand, $f(x) = x^2 + x$ is decreasing (can you see how you might verify this without graphing?); thus here the right-hand Riemann sum is associated with *inscribed* rectangles, and the left-hand Riemann sum is associated with *circumscribed* rectangles.
2. Observe that the values of $LHSum$, $MPSum$ and $RHSum$ are not necessarily the same for any finite n , but they get closer and closer to each other as n gets larger and larger, and their limits (as n goes to ∞) are *all* the same.
3. In addition to the command `sum(E(i), i=1..n)` there is a command `Sum(E(i), i=1..n)`. The difference between `sum` and `Sum` is that the former automatically expands and simplifies the sum, while the later does not: after using the later, you must expand and simplify it yourself if you want it done. (The value of the later command is that the former does not always work, so if you want to do your own simplification, you can. Also the latter can be useful for "pretty printing" output.)