

Project 4.9c: Newton's Method

Objective

To get some experience applying Newton's Method using Maple.

Narrative

If you have not already done so, read Section 4.9 in the text. There you will learn Newton's Method for finding an approximation to a root x of $f(x) = 0$ (or a *zero* of f) given an initial approximation x_1 to x . This method uses the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots,$$

x_1 being given, to obtain successive approximations x_2, x_3, \dots , to x .

Task

- a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands apply Newton's Method to finding the root of $f(x) = 8x^3 - 6x + 1 = 0$ given an initial estimate of $x_1 = 1.0$. The effect of each command is described in the right-hand column for your reference. Note that Maple uses brackets $[]$ to denote subscripts.

> # Your name, today's date	
> # Project 4.9c: Newton's Method	
> # Task 1	
> restart;	Clear Maple's memory.
> f := x -> 8*x^3-6*x+1;	Define f .
> plot(f(x), x=-2..2, y=-2..4);	Plot the graph of f .
> g := x -> x - f(x)/D(f)(x);	Let $g(x) = x - \frac{f(x)}{f'(x)}$.
> x[1] := 1.0;	Let $x_1 = 1.0$. (Make sure you type 1.0, not 1: it makes a difference to Maple!
> for n from 1 to 6 do x[n+1] := g(x[n]) end do;	This computes $x_{n+1} = g(x_n)$ for $n = 1, 2, \dots$ (Since $g(x) = x - \frac{f(x)}{f'(x)}$, it follows that $x_{n+1} = g(x_n)$.)

At this point, make a hard-copy of your typed input and Maple's responses. Then ...

- To 8 decimal places of accuracy, what number x_∞ does it appear that x_n is converging to?
 - Plot and label the point P on the x -axis whose x -coordinate is x_∞ .
- Use Newton's Method to estimate the solutions to the equation $\sin x = x^2 - 1$ by proceeding as follows:
 - Let $f(x) = \sin x - (x^2 - 1)$ and plot the graph of f for $x \in [-5, 5]$ and $y \in [-10, 2]$. (The graph will help you estimate the roots of $f(x) = 0$.)
 - Estimate each zero of $f(x)$ by a number x_1 , and iterate Newton's Method enough times to obtain an approximation to the root to 8 decimal places of accuracy. (*Note:* Your initial estimate of each root

must be close enough to the root for Newton's Method to converge: if it's not close enough, the values x_2, x_3, \dots might not converge to the value you're interested in!

Complete this task by again making a hard-copy of your typed input and Maple's responses (both text and graphics).

Your lab report will be the hard copies of your typed input and Maple's responses (both text and graphics).