Math 163, Fall 2001 **Due Date:**____________ **Name(s):** ________________________

Project 4.5b: Graphing Functions

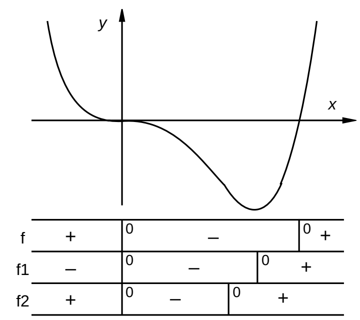
Objective

To investigate how we can relate where a function f and its first and second derivatives are positive, negative, zero, and do not exist, and the graph of f.

Narrative

The text provides one mechanism for gathering information to be used in graphing a function. In this project we present another.

To record where a function f and its first and second derivatives $f' = f1$ and $f'' = f2$, respectively, are positive, negative, zero, and do not exist, we use three "recording strips" below the graph of f as illustrated in the figure at the right; here $f(x) = (x^4 - 4x^3)/10$. (See Example 6, p. 245 of the text.) Study this figure carefully. Note how the vertical bars are drawn where f', f'', and f''' are 0 or do not exist, and the spaces between the vertical bars are labelled $+$ or $-$ depending on the sign of f', f'', and f''' over those intervals.



Tasks

1. Type the command lines below into Maple in the order in which they are listed. They initialize Maple for this project.

```
> # Your name, today’s date
> # Project 4.5b:  Graphing Functions
> restart;
> with(plots):
```

2. Type the command lines below into Maple in the order in which they are listed. They produce a graph of the function $f(x) = 4x/(1 + x^2)$, and three recording strips below the graph of f.

```
> # Task 2
> f := x -> 4*x/(1+x^2);
> plot0 := plot({-6,-5,-4,-3,0},x=-6..6,y=-6..3,color=black):
> plot1 := textplot({[-6,-3.5,´f´],[-6,-4.5,´f1´],[-6,-5.5,´f2´]}):
> plot2 := plot(f(x),x=-6..6):
> display({plot0,plot1,plot2});
```

3. a) Type the command lines below into Maple in the order in which they are listed. They determine where $f(x) = 3x^4 - 16x^3 + 18x^2$ and its first and second derivatives are positive, negative, zero, and do not exist; this information will be used later in this project.

```
> # Task 3
> f := x -> 3*x^4-16*x^3+18*x^2;
> evalf(solve(f(x)=0,x));
> f1 := D(f);
> evalf(solve(f1(x)=0,x));
> f2 := D(f1);
> evalf(solve(f2(x)=0,x));
```

b) Type the command line below into Maple. It produces an empty graph and three recording strips.

```
> display({plot0,plot1});
```

At this time, make a hard-copy of your input and Maple's responses. Then, ...

4. Fill in the recording strips on the graphic you produced in Task 2 using the graph of f in that graphic as a guide.

5. a) Fill in the recording strips on the graphic you produced in Task 3(b) using the information you computed in Task 3(a) as a guide. (You will need to test the values of f, f', and f'' between their respective zeroes to determine where they are positive and negative.)

   b) Use the information you recorded in (a) to sketch the graph of f in the space provided.

Comments

To find the intercepts and vertical asymptotes of the graph of f we need to find the values of x for which $f(x) = 0$ and $f(x)$ does not exist, to find the (possible) critical values of f we need to find the values of x for which $f'(x) = 0$ and $f''(x)$ does not exist, and to find the (possible) inflection points of the graph of f, we need to find the values of x for which $f''(x) = 0$ and $f''(x)$ does not exist. Since for every polynomial function f the values of $f(x)$, $f'(x)$, and $f''(x)$ exist for all x, in the above problem we only had to use information about where $f(x) = 0$, $f'(x) = 0$, and $f''(x) = 0$. If, for a function in a more general class of functions, $f(x)$, $f'(x)$, and/or $f''(x)$ did not exist for certain values of x, we would have to find these values and label vertical bars in recording strips with $\pm\infty$. (See Extra Project 4.5c.)