Math 163, Fall 2001 **Due Date:**____________ **Name(s):** ________________________

Project 4.2: The Mean Value Theorem

Objective

To illustrate the Mean Value Theorem.

Narrative

If you have not already done so, read Section 4.2 of the text. One of the key ideas you should take away from this section is that the Mean Value Theorem implies that if f is continuous on $[a, b]$ and differentiable on (a, b), then there is at least one value of $c \in (a, b)$ for which

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed.

`> # Your name, today's date`	
`> # Project 4.2:  The Mean Value Theorem`	
`> restart;`	Clear Maple's memory.
`> f := x -> x^4-16*x^3+92*x^2-224*x+200;`	Let $f(x) = x^4 - 16x^3 + 92x^2 - 224x + 200$.
`> plot(f(x),x=0..8,y=0..25);`	Plot the graph of f over the interval $[0, 8]$.
`> L := [[1.6,f(1.6)],[6.5,f(6.5)]];`	Let L be the line segment whose endpoints are $A(1.6, f(1.6))$ and $B(6.5, f(6.5))$.
`> plot({f(x),L},x=0..8,y=0..25);`	Plot L and the graph of f over the interval $[0, 8]$.

At this point, make a hard-copy of your typed input and Maple's responses. Then ...

b) using a straightedge, draw the tangent lines to the graph of f at those points whose x-coordinates are between $x = 1.6$ and $x = 6.5$, that are parallel to L, and

c) using a straightedge, drop perpendiculars from the points of tangency of the tangents you drew in part (b), to the x-axis, and estimate and label the values of c for which

$$f'(c) = \frac{f(6.5) - f(1.6)}{6.5 - 1.6}.$$

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

The Mean Value Theorem plays a very important role in Calculus. From it, for example, follow the facts that:

1. a function f is increasing over an open interval I if and only if $f'(x) > 0$ for each $x \in I$, and f is decreasing over I if and only if $f'(x) < 0$ for each $x \in I$, and
2. if $D_x(f(x)) = D_x(g(x))$ then $f(x) = g(x) + C$ for some constant C.

The first of these facts is important in applying differentiation to curve sketching. The second is important in proving and applying the Fundamental Theorem of Calculus!