

Project 3.10: Linear Approximation and Differentials

Objective

To illustrate the connection between the linear approximation of a function and its differential.

Narrative

If you have not already done so, read Section 3.10 of the text.

Linear approximations and differentials are important since they allow us to estimate the values of functions that are difficult or even impossible to evaluate. In this project we illustrate this by using differentials to approximate the value of $f(x) = \sqrt{2x}/(1-x)$ when $x = 0.7$.

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed.

> # Your name, today's date	
> # Project 3.10: Linear Approximation and Differentials	
> restart;	Clear Maple's memory.
> f := x -> sqrt(2*x)/(1-x);	Let $f(x) = \frac{\sqrt{2x}}{1-x}$.
> f(0.5);	Evaluate $f(x)$ when $x = 0.5$.
> df := (x,dx) -> (D(f)(x))*dx;	Let df be the differential of f .
> df(0.5,0.2);	Evaluate $df(x,dx)$ when $x = 0.5$ and $dx = 0.2$.
> with(student):	Use the <code>student</code> package to ...
> showtangent(f(x),x=0.5,x=0.1..0.9,y=0..5);	plot the graph of $f(x)$ near $x = 0.5$, and the tangent to the graph of f at $P(0.5, f(0.5))$.

At this point, make a hard-copy of your typed input and Maple's responses. Then ...

- b) draw by hand the horizontal line whose equation is $y = 2$, and the vertical lines whose equations are $x = 0.5$ and $x = 0.7$, and label each with its equation,
- c) clearly identify and label by hand the segments whose lengths are $\Delta y(0.5, 0.2)$ and $dy(0.5, 0.2)$, and
- d) using the values of $f(0.5)$ and $dy(0.5, 0.2)$ you computed above, estimate the value of $f(0.7)$.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics), and your approximation.

Comments

The actual value of $f(0.7)$ to 4 decimal places is 3.9441. While there is certainly some error in the approximation you made in this project, your approximation should still be fairly good considering the computations you had to perform (vs. the complexity of computing $f(0.7) = \sqrt{1.4}/0.3$ without using any computational devices other than a pencil and piece of paper). We will say considerably more about approximations later in MATH 164.

(The example used in this project is, to some extent, extreme: the relative error is large; we used this example, however, since the associated graphic is easy to work with. In many cases, differentials do a much better job of approximating: the relative error is small; "unfortunately" in these cases the associated graphic does not lend itself to visualization.)