C: ANSWERS TO SELECTED PROBLEMS

Chapter 4.1, Basic Concepts

1. $\Pr[7] = \frac{6}{36} = \frac{1}{6}$.
3. $\Pr[HHH] = \frac{1}{8}$.
5. $\Pr[T] = 1 - \Pr[H] = 1 - \frac{2}{3} = \frac{1}{3}$.
7. $\Pr[8'] = 1 - \Pr[8] = 1 - \frac{5}{36} = \frac{31}{36}$.
9. $\Pr[A \cup B] = 0.75 + 0.30 - 0.25 = 0.80$.
11. $\Pr[A \cap B] = 0.3 + 0.8 - 0.9 = 0.2$.
13. $\Pr[\#$ will occur at least once in 3 throws$] = 1 - \Pr[\#$ will not occur in 3$] = 1 - \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \approx 0.42$.
15. [a] Pr[at most 3 tails] $= 1 - \Pr[4 \text{ tails}] = 1 - \frac{1}{16} = \frac{15}{16}$, [b] 15:1, [c] 1:15.
17. $\Pr[12 \text{ or } 20] = \frac{6}{36} = \frac{1}{6}$.
19. $\Pr[A \cup B] = 0.55 + 0.25 - 0 = 0.80$.
21. $\Pr[C] = 1 - \Pr[A] - \Pr[B] = 1 - 0.15 - 0.45 = 0.40$.
23. $\Pr[\text{at least 1B and 1W}] = \dfrac{n(\text{at least 1B and 1W})}{n(S)} = \dfrac{C(6,1) \cdot C(4,2) + C(6,2) \cdot C(4,1)}{C(10,3)} = \dfrac{96}{120} = \dfrac{4}{5}$.
25. Pick 1 of 4 suits, then pick 5 cards in that suit,
$\Pr[\text{same suit}] = \dfrac{n(\text{same suit})}{n(S)} = \dfrac{C(4,1) \cdot C(13,5)}{C(52,5)} = \dfrac{5{,}148}{2{,}598{,}960}$.
27. Pick 1 of the 4 suits, there are 9 such sequences in each suit,
$\Pr[\text{straight flush}] = \dfrac{n(\text{straight})}{n(S)} = \dfrac{C(4,1) \cdot 9}{C(52,5)} = \dfrac{36}{2{,}598{,}960}$.
29. $\Pr[\text{B before C and D before E}] = \Pr[\text{B before C}] \cdot \Pr[\text{D before E}] = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

Chapter 4.2, Conditional Probability and Independence

1. [a] $\frac{1}{3}$, [b] 0.4.
3. $\Pr[\text{one die is 5} \mid \text{sum is 8}] = \dfrac{\Pr[\text{one die is 5} \cap \text{sum is 8}]}{\Pr[\text{sum is 8}]} = \dfrac{\frac{2}{36}}{\frac{5}{36}} = \dfrac{2}{5}$.
5. $\Pr[A \cup B] = \Pr[A] + \Pr[B] - \Pr[A \cap B] = \Pr[A] + \Pr[B] - \Pr[A] \cdot \Pr[B] = 0.3 + 0.5 - 0.3 \cdot 0.5 = 0.65$.
7. [a] 0.21, [b] 0.79, [c] 0.09.
9. [a] 0.8, [b] 0.5, [c] 0.2, [d] 0.26.
11. 0.08.
13. [a] $\Pr[2\,G \mid \text{at least 1}\,B] = \dfrac{\Pr[2\,G \cap 1\,B]}{\Pr[\text{at least 1}\,B]} = \dfrac{\frac{3}{8}}{\frac{7}{8}} = \dfrac{3}{7}$, [b] $\frac{1}{4}$.
15. Not independent, 0.001, 9 times.
17. 0.42.
19. $\Pr[1\text{st}\,H \mid \text{at least 2}\,H] = \dfrac{\Pr[1\text{st}\,H \cap \text{at least 2}\,H]}{\Pr[\text{at least 2}\,H]} = \dfrac{\frac{3}{8}}{\frac{4}{8}} = \dfrac{3}{4}$.
21. [a] Pr[2R] = 0.05 + 0.03125 = 0.08125, [b] Pr[at least 1 R] = 0.56875.
23. $\Pr[4R \mid \text{at least 2}R] = \dfrac{\Pr[4R \cap \text{at least 2}R]}{\Pr[\text{at least 2}R]} = \dfrac{\frac{C(10,4)}{C(18,4)}}{\frac{C(10,2)C(8,2)+C(10,3)C(8,1)+C(10,4)}{C(18,4)}}$.
25. [a] 21/44, [b] 6/11.
27. [a] 4/52, [b] 13/51, [c] 16/2,652.
29. 0.4148.
31. [a] 0.20, [b] 0.2667, [c] 0.45.

Chapter 4.3, Bayes' Probability

1. $\Pr[\text{HIV} \mid \text{PR}] = \dfrac{\Pr(\text{HIV} \cap \text{PR})}{\Pr(\text{HIV} \cap \text{PR}) + \Pr(\text{Not HIV} \cap \text{PR})} = \dfrac{(0.004)(0.98)}{(0.004)(0.98) + (0.996)(0.09)} \approx 0.0419$.
3. $\Pr[\text{HIV} \mid \text{NR}] == \dfrac{(0.004)(0.02)}{(0.004)(0.02) + (0.996)(0.91)} \approx 0.0000883$.
5. [a] $\Pr[A \mid C] = \frac{0.10}{0.10+0.16} = \frac{5}{13}$, [b] $\Pr[B \mid C] = \frac{0.16}{0.10+0.16} = \frac{8}{13}$, [c] $\Pr[B \mid D] = \frac{0.24}{0.10+0.24} = \frac{12}{17}$, [d] 1.
7. $\frac{1}{3}$.
9. $\Pr[A \mid \text{Rotten}] = \frac{(0.5)(0.04)}{(0.5)(0.04)+(0.5)(0.06)} = 0.4$.
11. $\Pr[\text{Actually true} \mid \text{Identified as lie}] = 0.375.0.375$.

13. $\Pr[\text{Heads} \mid \text{White}] = \frac{20}{37}$.
15. $\Pr[\text{Man} \mid \text{Lunch}] = \frac{\frac{2}{3}\cdot\frac{1}{4}}{\frac{2}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{2}} = 0.5$.
17. $\Pr[\text{AAA} \mid \text{Defaulted}] = 0.05$.
19. $\Pr[\text{New program} \mid \text{Accident}] = \frac{2}{3}$.
21. $\Pr[\text{Two-tail coin} \mid \text{Tails}] = \frac{2}{3}$.
23. $\Pr[\text{Two-tailed urn} \mid \text{Tails}] = \frac{49}{89}$.
25. [a] $\frac{5}{16}$, [b] $\frac{1}{2}$, [c] $\frac{1}{8}$.
27. 1/3.

Chapter 4.4, Bernoulli Trials

1. 0.2373.
3. 0.1977.
5. 0.2254.
7. $1 - \Pr[\text{success on all 10}] = 1 - b(10;10,\frac{1}{6}) \approx 1$.
9. [a] $b(2;12,0.05) \approx 0.0988$, [b] $1 - b(0;12,0.05) - b(1;12,0.05) \approx 0.1183$.
11. $b(3;5,0.5) + b(4;5,0.5) + b(5;5,0.5) = 0.5$.
13. $1 - b(0;6,0.25) - b(1;6,0.25) \approx 0.466$.
15. 0.0026.
17. $b(2;4,0.25) \approx 0.2109$.
19. Find smallest n, $1 - b(0;n,0.5) > 0.9$, $n = 4$.
21. 0.1316.
23. [a] 0.1509, [b] 0.8814.
25. $\Pr[4 \text{ or } 5] \approx 0.4398$ is most likely, then $\Pr[> 5] \approx 0.3348$, and least likely $\Pr[< 4] \approx 0.2254$.
27. 14.
29. 4.

Chapter 4.5, Chapter Review

Mastery Quiz

1. [b], 2. [b], 3. [b], 4. [d], 5. [a], 6. [c], 7. [d], 8. [c], 9. [c], 10. [b]

Review

1. [a] .6, [b] .12, [c] .2, [d] 0.
3. [a] 3/8, [b] 3:5, [c] 5:3.
5. 0.375.
7. 0.6.
9. 3/13.
11. Pick 2 ranks, then 1 of 2 for triple, pick 3 cards, pick 2 cards,
$$\Pr[\text{Full House}] = \frac{C(13,2)\cdot C(2,1)\cdot C(4,3)\cdot C(4,2)}{C(52,5)}.$$
13. $\Pr[5 \text{ consecutive cards in same suit}] = \frac{C(4,1)\cdot 10}{C(52,5)}$.
15. 0.0698.
17. 0.2.
19. 20/21.
21. 0.7.
23. 0.1.
25. $\Pr[2\ M \mid \text{at least 1 } M] = \frac{\frac{C(5,2)}{C(8,2)}}{\frac{C(5,1)C(3,1)+C(5,2)C(3,0)}{C(8,2)}} = \frac{2}{5}$.
27. $b(13;15,0.5) + b(14;15,0.5) + b(15;15,0.5) \approx 0.0037$.
29. 1/120.
31. [a] 1/201 [b] 3/250,003 [c] 3/500,003.