C: ANSWERS TO SELECTED PROBLEMS

Chapter 3.1, Trees and Equally Likely Outcomes

1. *S* = {*HHH, HHT, HTH, THH, HTT, THT, TTH, TTT*}.
3. *S* = {1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 30, 36}. Remember, repeated elements are rostered only once in a set.
5. $6 \cdot 2 = 12$, *S* = {*1H, 1T, 2H, 2T, 3H, 3T, 4H, 4T, 5H, 5T, 6H, 6T*}.
7. $3 \cdot 2 \cdot 1 = 6$, *S* = {*ABC, ACB, BAC, BCA, CAB, CBA*}.
9. There will be three different outcomes on the tree diagram, *S* = {*RB, BB, BR*}.
11. $3 \cdot 2 \cdot 1 = 6$
13. 5,040.
15. $5! = 120$.
17. [a] $3,628,800 - 120 = 3,628,680$; [b] $5! = 120$; [c] $2,730$; [d] 306.
19. 4! = 24.
21. $20 \cdot 19 \cdot 18 = 6,840$.
23. Each of the 5 persons will shake hands with 4 other people. $5 \cdot 4 = 20$. However, this includes a second handshake for each pair of people, so we divide this number in half. There are 10.
25. $4 \cdot 3 \cdot 2 \cdot 1 = 24$.
27. $1 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 50,000,000$.
29. $1 \cdot 4 \cdot 3 + 1 \cdot 4 \cdot 2 + 1 \cdot 4 \cdot 3 + 1 \cdot 4 \cdot 2 + 1 \cdot 4 \cdot 3 + 1 \cdot 4 \cdot 2 = 60$.
31. $2^{10} \cdot 5^{10} = 10^{10}$.

Chapter 3.2, Permutations

1. [a] 120, [b] 720, [c] 5, [d] 1.
3. *P*(20, 3) = 6,840.
5. *P*(7, 4) = 840.
7. There are 11 letters of which the *M*, *A*, and *T* are each repeated twice, $\frac{11!}{2! \cdot 2! \cdot 2!} = 4,989,600$.
9. $7 \cdot 7 \cdot 7 \cdot 7 = 7^4 = 2,401$.
11. Fixing one seat at the table converts the problem to 14 seats (1 vacant) in a row, or 13! ways.
13. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 1 = 10,000$.
15. Start with (8 - 1)! ways to place 8 keys on a ring (circular permutations) then multiply by 2 for each key (each key could face up or down). Therefore, $(8-1)! \cdot 2^8 = 1,290,240$.
17. $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 1,000,000$.
19. $P(6,4) = 360$.
21. $26 \cdot 25 \cdot 4 \cdot 4 = 10,400$.
23. $4 \cdot 8 \cdot 7 \cdot 3 \cdot 6 \cdot 5 \cdot 2 \cdot 4 \cdot 3 \cdot 1 \cdot 2 \cdot 1 = 967,680$.
25. Start with the total number of ways with no restrictions, $5 \cdot 4 \cdot 3$, then subtract the number of ways with all evens, 0 (there is only 2 even elements); and subtract the number of ways for all odds, $3 \cdot 2 \cdot 1$. That is, 60 – 0 – 6 = 54.
27. 8 games are required.
29. The row could begin with a boy or a girl, 2 choices. Then, *P*(5,5) ways to seat the boys and *P*(5,5) ways to seat the girls. $2 \cdot P(5,5) \cdot P(5,5) = 2 \cdot 120 \cdot 120 = 28,800$.
31. $\frac{15!}{(3!)^5} = 168,168,000$.

Chapter 3.3, Combinations

1. [a] 1, [b] 210, [c] 1, [d] 6.
3. *C*(20, 3) = 1,140.
5. *C*(52, 5) = 2,598,960.
7. $C(3,1) \cdot C(1,1) \cdot C(2,1) = 6$.
9. Read across the 6 th row of the triangle, [a] 1, [c] 10, [e] 5.

11. Either 1 B and 2 non-B or 2B and 1 non-B will satisfy, $C(2,1)\cdot C(9,2)+C(2,2)\cdot C(9,1)=81$.
13. Need to pick 2 kinds, then pick which of the 2 will be 4 of kind, then pick 1 card of the other kind. $C(13,2)\cdot C(2,1)\cdot C(4,4)\cdot C(4,1)=624$.
15. $P(31, 2) = 930$.
17. $2^5 = 32$.
19. $C(20,2)\cdot C(15,2)=190\cdot 105=19,950$.
21. [a] $C(20,3)=1,140$.
    [b] (2F and 1M) or (3F and 0M): $C(10,2)\cdot C(10,1)+C(10,3)\cdot C(10,0)=45\cdot 10+120\cdot 1=570$.
    [c] Find the complement to part b, $1,140-570=570$.
23. [a] Pick a suit, then pick 5 cards in suit, $C(4,1)\cdot C(13,5)=5,148$.
    [b] Pick 3 clubs, then pick 2 non- clubs, $C(13,3)\cdot C(39,2)=211,926$.
25. $C(30, 5) = 142,506$.
27. [a] $C(8, 4) = 70$, [b] $C(2,2)\cdot C(8,2)=28$.
29. To pick the 8 in favor (or 4 against) is $C(12,8)=C(12,4)=495$.

Chapter 3.4, Chapter Review

Mastery Quiz

1. [a], 2. [b], 3. [a], 4. [a], 5. [b], 6. [a], 7. [b], 8. [b], 9. [c], 10. [c]

Review

1. [a] 336, [b] 720, [c] 15, [d] 362,880, [e] 36.
3. [a] $P(20, 3) = 6,840$ [b] $C(20, 3) = 1,140$.
5. $99\cdot 26\cdot 10\cdot 10\cdot 10\cdot 10=25,740,000$.
7. $\dfrac{40!}{6!\cdot 6!\cdot 8!\cdot 8!\cdot 12!}\approx 2.02\times 10^{24}$.
9. Cannot have 1 M and 3 F because there are only 2 F, thus 2 M and 2 F or 3 M and 1 F, $C(6,2)\cdot C(2,2)+C(6,3)\cdot C(2,1)=55$.
11. First select toppings, then crust, then drink, $C(12,3)\cdot 2\cdot C(6,1)=2,640$.
13. $5\cdot 4\cdot 9\cdot 9\cdot 9\cdot 9=131,220$.
15. $\dfrac{11!}{4!\cdot 4!\cdot 2!}=34,650$.
17. [a] $C(12,3)\cdot C(10,3)=26,400$.
    [b] Use the complement of 0 boys and 6 girls: $C(22,6)-C(12,0)\cdot C(10,6)=74,403$.
    [c] Select 6 from 22 in combination, then select 2 from these 6 in permutation: $C(22,6)\cdot P(6,2)$.
19. Construct a chart, 5 ways.
21. $(8-1)!=5,040$.
23. Row starts with men or with women, $P(4,4)\cdot P(4,4)+P(4,4)\cdot P(4,4)=1,152$.
25. $P(10,4)\cdot P(8,2)=282,240$.
27. $\dfrac{n\cdot(n-1)}{2}=105$, where $n=15$.
29. $C(31,3)\cdot 4\cdot 2=35,960$.
31. [a] Number of distinguishable arrangements of 12 people is 12!, with pairing of 2 people per room being indistinguishable is $\dfrac{12!}{2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!\cdot 2!}=7,484,400$.
    [b] Pick 3 of the 6 rooms for the men, arrange the 6 men into 3 groups of 2, then arrange the 6 women into 3 groups of 2. $P(6,3)\cdot\dfrac{6!}{2!\cdot 2!\cdot 2!}\cdot\dfrac{6!}{2!\cdot 2!\cdot 2!}=972,000$.
33. $2\cdot P(4,4)=48$.