Lesson #7: Divergence & Curl of E, The Dirac Delta Function **Name: ______________________________**

Here are two universal truths for electrostatics:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_o}$$ (Gauss's law in differential form, discussed in class)

$$\vec{\nabla} \times \vec{E} = 0$$ (No name, but always true in electrostatics! More next lesson…)

These equations mean that there are set rules for the existence of electric fields. In other words, not everything is possible and not every vector field can represent a <u>real</u> electric field.

Check the following possible electric fields and determine if they are valid. If so, determine the charge density (ρ) that produced them. In each case, "*k*" is just a constant. **Show your work (i.e. the math).**

1) $\vec{E} = k[xy\,\hat{x} \;-\; 2y\,\hat{y} \;+\; 3xz\,\hat{z}]$ This field is: **valid / not valid**

2) $\vec{E} = k\,[y^2\hat{x} \;+\; (2xy \;+\; z^2)\,\hat{y} + 2yz\,\hat{z}]$ This field is: **valid / not valid**

3) $\vec{E} = k\,[sz\,\hat{s} \;+\; z\hat{\phi} + s\phi\,\hat{z}]$ *(cylindrical coordinates)* This field is: **valid / not valid**

4) $\vec{E} = k\,r^4\,\hat{r}$ *(spherical coordinates)* This field is: **valid / not valid**

Read Section 1.5 and study example 1.14. Then evaluate the integrals below.

5) $\int_2^6 \left(3x^2 - 2x - 1\right)\delta(x-3)\,dx \quad =$

6) $\int_0^3 x^3\;\delta(x+1)\,dx \quad =$

7) $\int_{all\ space} \left(r^2 + \vec{r}\cdot\vec{a} + a^2\right)\delta(\vec{r}-\vec{a})\,d\tau \quad =$ (where $\vec{a} = 2\hat{x} - \hat{y} + 3\hat{z}$)