Lesson #30: Divergence and Curl of B Name: ____________________________

Study section 5.3, then answer the following questions.

The divergence and curl of electrostatic and magnetostatic fields are not arbitrary, but are given by the equations

$$\vec{\nabla}\cdot\vec{\mathbf{E}}=\frac{\rho}{\varepsilon_0} \qquad \vec{\nabla}\times\vec{\mathbf{E}}=0$$

$$\vec{\nabla}\cdot\vec{\mathbf{B}}=0 \qquad \vec{\nabla}\times\vec{\mathbf{B}}=\mu_0\,\vec{\mathbf{J}}$$

Together, these four equations are referred to as Maxwell's equations for static fields. Maxwell's four equations plus the Lorentz force law represent a complete formulation of electromagnetism, aside from the constituent equations required for dielectrics and magnetic materials. Maxwell's equations will require modification when the fields are time-dependent (next semester!).

1. The magnetic field of a long, straight wire carrying a current *I* in the *z* direction is $\vec{\mathbf{B}}=\frac{\mu_0 I}{2\pi s}\hat{\phi}$. Show that both the divergence and curl of the field are as expected. Show all your work on this—if certain terms go to zero in the div or curl, show/explain why.

2. What does the equation $\vec{\nabla}\cdot\vec{\mathbf{B}}=0$ physically mean? Explain in a few sentences the reasoning behind your answer.

3. Starting from the curl equation, $\vec{\nabla}\times\vec{\mathbf{B}}=\mu_0\,\vec{\mathbf{J}}$, derive the integral from of Ampere's law, equation 5.55 in the text. Again, show every step of the mathematics.