Q1 A 50 μF capacitor stores 0.01 Joules of energy. A second capacitor of value 20 μF carries the same voltage as the first. Please find the charge on the second capacitor.

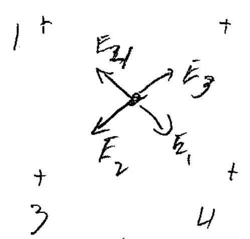
a) 2×10^{-4} C
b) 4×10^{-4} C
c) 6×10^{-4} C
d) 8×10^{-4} C
e) None of the above

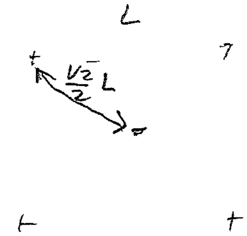
$U = \frac{1}{2}CV^2 \Rightarrow V = \sqrt{\frac{2U}{C}} = 20\,V$

$Q = CV = \boxed{4 \times 10^{-4}\,C}$

Q2 Identical positive charges Q are placed at the corners of a square of side L. Assume we use the convention that V = 0 at infinity. E is the magnitude of the electric field, V is the electric potential. Please indicate which of the following statements is true at the center of the square.

b) $E > 0,\ V > 0$
c) $E > 0,\ V = 0$
c) $E > 0,\ V < 0$
d) $E = 0,\ V > 0$
e) $E = 0,\ V = 0$
f) $E = 0,\ V < 0$
g) None of the above





$\vec{E}$'s ADD TO 0.

$U = \frac{kQ}{\frac{\sqrt{2}}{2}L}$ FOR EACH POSITIVE

P1 A proton is released from rest in a uniform electric field of magnitude 75,000 V/m. Please find

a) The electric force on the proton.
b) The proton's kinetic energy after it has travelled 5 cm.
c) The surface charge density σ that must be placed on a infinite plane to produce the field.

$F = qE = (1.602 \times 10^{-19})(75{,}000) = \boxed{1.2 \times 10^{-14}\,N}$

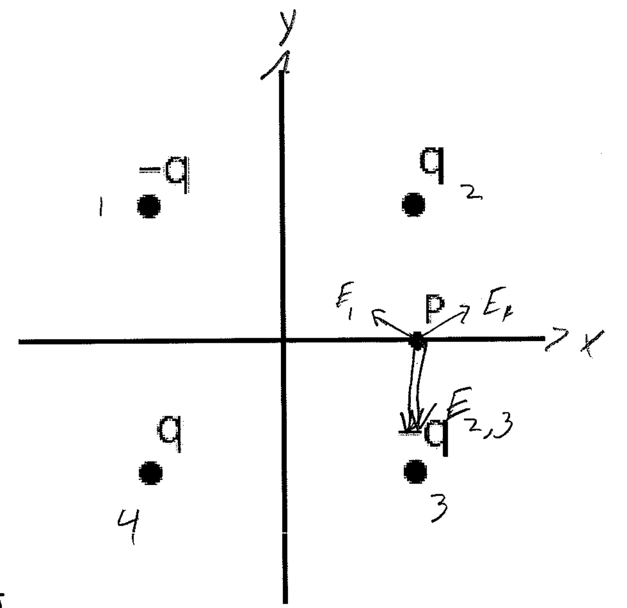
$W = K_f - K_i^{\,0} \Rightarrow K_f = Fx = (1.2 \times 10^{-14})(0.05)$

$= \boxed{6 \times 10^{-16}\,J}$

$E = \frac{\sigma}{2\epsilon_0}$ FOR AN INFINITE PLANE

$\Rightarrow \sigma = 2\epsilon_0 E = \boxed{1.33 \times 10^{-6}\,\frac{C}{m^2}}$

P2 Two positive charges and two negative charges, all of magnitude 4 μC, are placed at four corners of a square with side 2 m centered at the origin. The situation is shown in the figure.



Please answer each of the following questions.

a) What is the direction of the electric field at point P located 1 m from the origin on the positive x-axis?

b) What is the total potential energy of the system of four charges?

c) A charge Q =1 μC is placed at point P. What is the net force on Q?

a) FIELD IN $-\hat{j}$ DIRECTION

b) $U_T = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$= \frac{-kq^2}{2} + \frac{kq^2}{2\sqrt{2}} + \frac{-kq^2}{2} + \frac{-kq^2}{2} + \frac{kq^2}{2\sqrt{2}} + \frac{-kq^2}{2}$$

$$= kq^2\left(\frac{-4}{2} + \frac{2}{2\sqrt{2}}\right) = kq^2\left(\frac{1}{\sqrt{2}} - 2\right) = -0.186\ J$$

c) $\vec{F} = Q\vec{E}$ FIND $\vec{E}$ FIRST

USE $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$

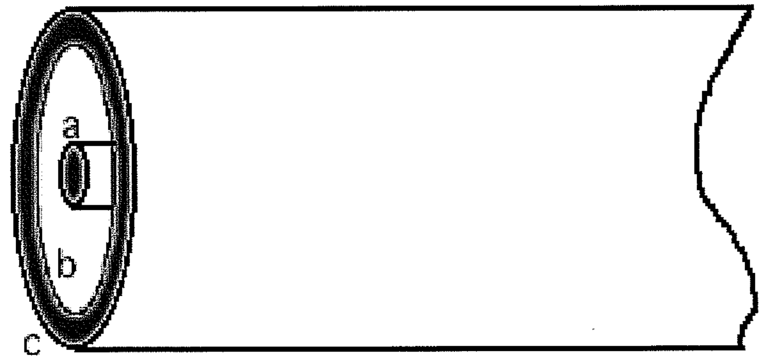
$$= \frac{-kq}{(2^2+1^2)}\left(\frac{2\hat{i}-\hat{j}}{\sqrt{5}}\right) + \frac{kq}{1^2}(-\hat{j}) + \frac{-kq}{1^2}(\hat{j}) + \frac{-kq}{1^2+2^2}\left(\frac{2\hat{i}+\hat{j}}{\sqrt{5}}\right)$$

$$= \frac{kq}{5\sqrt{5}}(2\hat{j}) + \frac{kq}{1}(-2\hat{j}) = 2kq\left(1 - \frac{1}{5\sqrt{5}}\right)\hat{j}$$

$$= -6.56 \times 10^4 \frac{N}{C}$$

SO $\vec{F} = q\vec{E} = 6.56 \times 10^{-2}\ N$

P3 A long conducting wire of radius a is surrounded by a conducting coaxial cylinder of inner radius b and outer radius c. The wire carries a net linear charge density λ and the cylinder carries a net linear charge density 2λ.



Please answer each of the following questions.

a) What is the linear charge density on each of the following surfaces?

(i) $r = a$ $\boxed{\lambda}$ (ALL CHARGE ON INNER CONDUCTOR TO SURFACE)

(ii) $r = b$ $= \boxed{-\lambda}$ (TO FORCE $\vec{E} = 0$ INSIDE OUTER CONDUCTOR)

(iii) $r = c$ $= \boxed{3\lambda}$ (SO THAT NET CHARGE ON OUTER CONDUCTOR IS 2λ)

b) What is the magnitude of the electric field in each of the following regions?

(i) $r < a$ $E = 0$ INSIDE CONDUCTOR

(ii) $b < r < c$ $E = 0$ INSIDE CONDUCTOR.

(iii) $r > c$ USE GAUSS'S LAW, OR RECALL THAT $\vec{E}$ OF AN INFINITE CYLINDER IS IDENTICAL TO THAT OF AN INFINITE WIRE

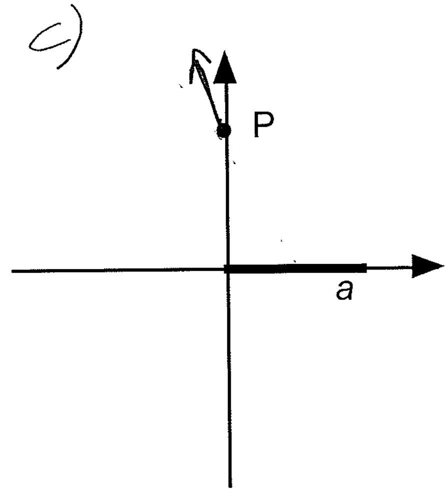
$$\Phi = \frac{Q_{enc}}{\epsilon_0}$$

USE GAUSSIAN CYLINDER OF RADIUS r LENGTH ℓ

$$E(2\pi r \ell) = \frac{3\lambda\ell}{\epsilon_0} \qquad \boxed{E = \frac{3\lambda}{2\pi\epsilon_0 r}}$$

DIRECTION IS RADIALLY OUT

P4 Charge Q is distributed uniformly over a thin rod of length a, which lies along the poitive x-axis as shown.



Please answer each of the following questions.

a) Is the rod an insulator or a conductor?

b) What is the linear charge density on the rod?

c) Draw an arrow on the figure indicating the approximate direction of the electric field at point P located at $(0, y)$.

d) Determine the x-component of the electric field at point P. You should find one of these integrals useful:

$$\int \frac{x\,dx}{\left(x^2+b^2\right)^{\frac{3}{2}}} = -\frac{1}{\sqrt{x^2+b^2}} \qquad \int \frac{dx}{x^2+b^2} = \frac{1}{b}\arctan\left(\frac{x}{b}\right) \qquad \int \frac{dx}{\left(x^2+b^2\right)^{\frac{3}{2}}} = \frac{1}{b^2}\frac{x}{\sqrt{x^2+b^2}}$$

a) ONLY **INSULATOR** CAN SUPPORT UNIFORM CHARGE DISTRIBUTION

b) $\boxed{\lambda = \frac{Q}{a}}$ $\qquad \frac{\text{TOTAL CHARGE}}{\text{TOTAL LENGTH}}$

c) SEE FIGURE

d) $\vec{E} = \int_{\text{ALL CHARGE}} k\frac{dq}{r^2}\hat{r}$ $\qquad$ PARAMETRIZE OVER X

$$\vec{E} = \int_0^a k\frac{\lambda dx}{x^2+y^2}\frac{(-x\hat{\imath}+y\hat{\jmath})}{\sqrt{x^2+y^2}}$$

$\hat{\imath}$ TERM GIVES E_x

$$E_x = \frac{-kQ}{a}\int_0^a \frac{x\,dx}{(x^2+y^2)^{\frac{3}{2}}} = -\left(\frac{kQ}{a}\right)\left(\frac{-1}{\sqrt{x^2+y^2}}\right)\Bigg|_0^a = \boxed{\frac{kQ}{a}\left(\frac{1}{\sqrt{a^2+y^2}} - \frac{1}{y}\right)}$$