Physics 251 Laboratory

Calculus Workshop

For the next three lab periods we will be reviewing the concept of density and learning the calculus techniques necessary to succeed in Physics 251. The first week will focus on linear, surface and volume densities. The second week will be a review of integral calculus applied to real physics problems with physical units. The third week will focus on the vector calculus required to calculate electric and magnetic fields. There will be pre-lab exercises on the web before each workshop.

Calculus Workshop 1

Pre-Lab: Please read and think about this handout before coming to the lab and don't forget the pre-lab exercises on the web.

Introduction

You have probably already been introduced to the concept of density, represented by scientists and engineers as the Greek letter rho (ρ). Density is commonly defined as mass per unit volume of a given substance. However, "density" can have many other meanings. Some other densities used in Phys 251 are charge density, current density, energy density, and number density. This lab will also help define densities besides "per volume", including "per area" and "per length."

Equipment/Supplies

1. A triple beam balance
2. A ruler
3. A rubber rectangle covered with pips.

Section 1

Surface mass density can be expressed as mass divided by area. We will use the Greek letter sigma (σ) to denote surface density. Writing out the formula, $\sigma_m = \frac{mass}{area}$, helps in deciding what information is required. The area can be expressed as $A = \text{length} \times \text{width}$, and therefore $\sigma_m = \frac{mass}{L \times W}$. You have a rubber rectangle with 'bumps' on its surface. These 'bumps' are called pips. We are going to calculate the surface number density of these pips. Surface number density, σ_n, is the number of pips divided the area of the rectangle.

1. What would be the surface number density of your rectangle?
2. What would the surface charge density, σ_q, be, if each pip carried a charge of $2\mu C$?

Note: In 251, we talk about charge more often than anything else, so we often use σ by itself, with no subscript, to denote surface charge density.

Section 2

Rolling the rectangle into a long cylinder, and using the previously calculated σ_n, we can find the linear number density, λ_n, which is the total number of pips per unit centimeter along the cylinder. Note that we use the Greek letter lambda (λ for linear densities. So far, we have **L**inear and **L**ambda; **S**urface and **S**igma; **V**olume and **R**ho.

To calculate λ_n, we need to multiply our previously obtained surface number density, σ_n , by the area per unit length of the rectangle, in terms of the units:

$$\lambda_n = \sigma_n (\frac{cm^2}{cm})$$

The result is $\lambda_n = (\sigma_n)$(width of the rectangle). Please prove this and summarize your proof on the back page. Here are some additional exercises:

1. Calculate the linear number density, λ_n, for your cylinder.

2. Calculate the linear charge density for your cylinder (again, use 2 μC per pip).

3. If you unroll the cylinder does the rectangular piece still have a linear number density or linear charge density? Are they the same as for the cylinder?

4. Calculate the linear mass density of the cylinder.

5. If you roll the rectangle up tightly (so that its radius is small, and many of the pips are inside) is the linear mass density affected? Assume we had a rubber mat big enough to cover this room, and we rolled it up along its short direction. How much would it weigh? What would be its linear mass density?

6. If you unroll the cylinder, does the rectangular piece still have a linear mass density? Is it the same as for the cylinder?

7. Does the rectangular piece have a linear mass density measured in the perpendicular direction (along the short edge)? Is it the same as before?

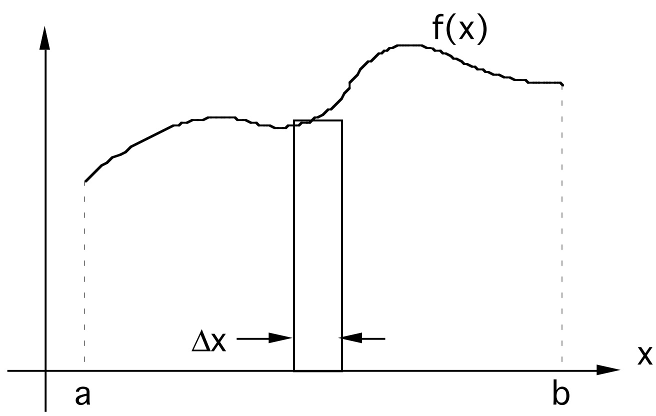
Calculus Workshop 2

Introduction

Last week, we explored the idea of density. Starting from the volume mass density we talked about the surface mass density, surface number density, linear mass density, etc. All of the densities we talked about were constants. This is not necessary, however, and we can still make good use of the idea of a density even if it isn't constant, that is, if it is a function of position. What we need is to use calculus. In the process, we will explore what an integral **really** means. This lab will introduce you to some basic concepts of integration and will ask you to apply these concepts to several physical problems.

Section 1

In calculus class, you learned that an integral gives the area under a curve. Your professor probably drew a diagram much like this one.



The area under the curve is approximately equal to the sum of the areas of a bunch of rectangles. Each rectangle has an area given by its width, Δx, times its length, $f(x)$. At each value of x, $f(x)$ has a different value, but the width Δx is constant. The mathematical expression equivalent to this is

$$A \approx \sum_{i=1}^{N} f_i(x)\Delta x .$$

The idea of an integral is that we take the rectangles to be infinitely narrow (infinitesimal) and we add up an infinite number of them. This eliminates the errors that are due to the flat tops of the rectangles not reproducing the curve perfectly. To indicate the "infinitely narrow" business, we change notation to

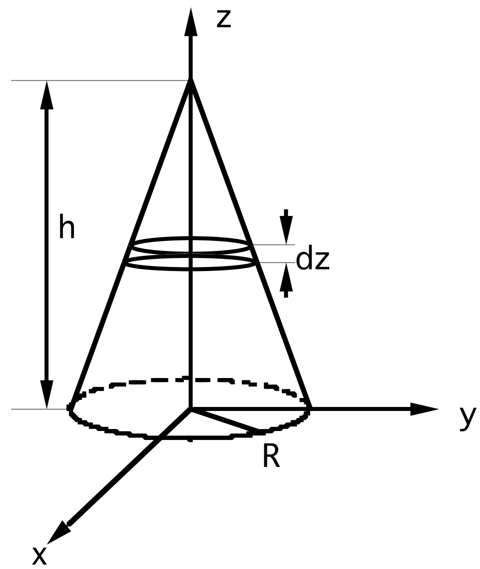
$$A = \int_a^b f(x)dx$$

The integral sign replaces the summation sign, the dx replaces Δx and the limits of integration replace the numbering of the rectangles. **In either case, a physicist must point something out: both f(x) and dx must have units of length (e.g. meters) for the integral to be an area.** In physics, we will use the idea of an integral to mean more than just an area. Anything that we want to "total up" that varies from place to place we will calculate using integrals. In the example above, we totaled up a the areas of a bunch of (infinitely narrow) rectangles.

However, we can also total up physical quantities besides areas. In class recently, we have totaled up the electric fields due to infinitesimal charges. The result in this case is the total electric field due to a distribution of charge. Symbolically:

$$\vec{E} = \int d\vec{E} = \int_{\text{charge}} \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \hat{r}$$

Section 2



Consider a cone of height h and radius R at its base. One way to calculate the volume of a cone is to add up volumes of horizontal slices. In this case, the integral is over volume elements: dV = Adz = πr^2dz. In order to do the integral, we need a formula for r(z), because the radius is a function of z. We can get this be noticing that the radius decreases linearly as a function of z from R at z = 0 to 0 at z = h.

$$r = R - \frac{R}{h} z$$

1. Set up and evaluate the integral for the volume of the cone, verify your answer with the known formula for the volume of a cone.

Now, what if it is the mass of the cone that we want rather than the volume. One way to find the mass would be to calculate the volume and multiply by the density (M = ρV). Another way would be to get the masses of the slices dM = ρdV and integrate those. Since ρ is constant it can be pulled out in front of the integral, and the result is the same (M =ρV).

But, what if the density isn't a constant?

In that case the old method M = ρV = $\rho\int dV$ doesn't work. However, the method I just suggested does. That is, we can still use

$$M = \int dm = \int \rho dV$$

As a concrete example, imagine that the cone above has a density that varies with z according to

$$\rho = C(h–z)$$

Where C is a constant. How do we interpret this? The density is zero right near the top of the cone, but it increases linearly as you go down, and it is ρ = Ch right near the bottom.

2. What must be the units of the constant C?
3. Set up and evaluate the integral for the mass of this cone

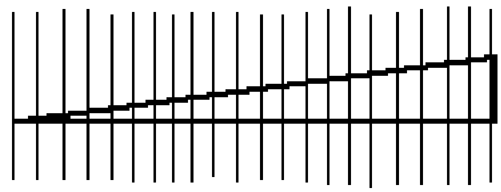
Note: now that you have the idea of variable densities, and the idea of surface densities, linear densities charge densities etc. it is not too far of a leap to imagine a variable linear charge density. Think about how would you figure out the E field of such a thing.

Section 3

1. You have been given a piece of rubber covered with pips in the shape of a long, thin right triangle. Last week, you calculated the linear mass density, λ_m of a cylinder. Now, explain how we can define a linear mass density for the triangle.

2. Measure the dimensions of your triangle, and **calculate** the linear mass density as a function of position, $\lambda_m(x)$, along the longer of the two legs (not the hypotenuse (useful constants from last week: $\sigma_m = 0.384$ (g/cm^2) $\sigma_n = 2.33$ (pips/cm^2))

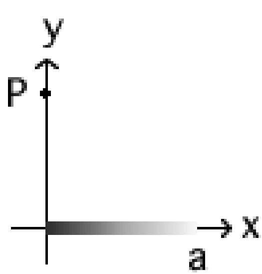
3. Below is data for one triangular piece that was cut up (as shown) and measured in pieces. Plot and this data in such a way as to be able to determine the mass per unit length.



4. Using the data, calculate the linear mass density as a function of position $\lambda_m(x)$

Piece #	Length (cm)	mass (g)
1	3.4	0.37
2	3.4	0.91
3	3	1.39
4	3	1.95
5	3	2.3
6	3	2.72
7	3	3.41
8	3	3.88
9	3	4.4
10	3	4.82
11	3	5.14
12	3	5.39
Totals	36.8	36.68

5. Try to write down a formula for the electric field at point P at (0,y) due to the object shown below. It has a linear charge density $\lambda = \lambda_0(a-x)$ spread over the x-axis between the origin and the point x = a.



Calculus Workshop 3

Introduction

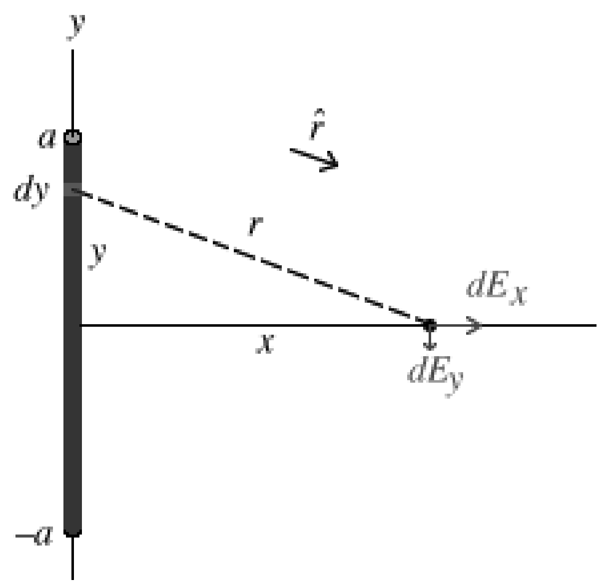
Last week, we had a quick introduction to integration. This was a review for most of you, although you may have found being able to do an integral is a very different skill from deciding when an integral must be done, and deciding which integral is needed. This week, we will go still further. We will discuss vector integrals. Simply put, these are integrals in which the integrand involves one or more vectors. If the integrand is a vector (including a scalar times a vector) then the result will be a vector too. If the integrand is a scalar (for instance, a dot product of two vectors) then the result will be a scalar. In either case, though, the lesson learned last week still stands. An integral is a "total" found by adding up infinitesimal pieces. If we add up little vectors, we get a vector, etc.

Section 1

The first type of vector integral we talked about in class was the kind where the integrand is a scalar times a vector. This describes integrals like

$$\vec{E} = \int_{\text{all charge}} \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}\,.$$

In this case, $dq/4\pi\varepsilon_0 r^2$ is a scalar, and $\hat{r}$ is a vector, so the whole integrand is a vector. A good example of a problem that requires this treatment is calculating the electric field due to a linear charge distribution, as in Example 22-11 in the text:



Positive electric charge Q is distributed uniformly along a line with length $2a$, lying along the y-axis between $y = -a$ and $y = +a$ as in the figure. Find the electric field at point P on the x-axis at a distance x from the origin.

In this lab, we will investigate what the vector integral means by approximating it as a sum of 10 small vectors. To be more specific, let's give some exact values: $Q = 20$ nC, $a = 5$ cm, and $x = 8$ cm. Using these values, please calculate the electric field at P using the exact formula,

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{2ax}\frac{1}{\sqrt{\left(x^2/a^2\right)+1}}\hat{i}\,.$$

Please write the value you get on the results page.

To approximate the electric field, we will calculate the electric field due to each of ten pieces of the line of charge. The table on the results page will guide you through this process. (Hint: $k = 1/4\pi\varepsilon_0 = 9 \times 10^9$). Please work to fill out the table, and answer the questions that follow it.

Section 2

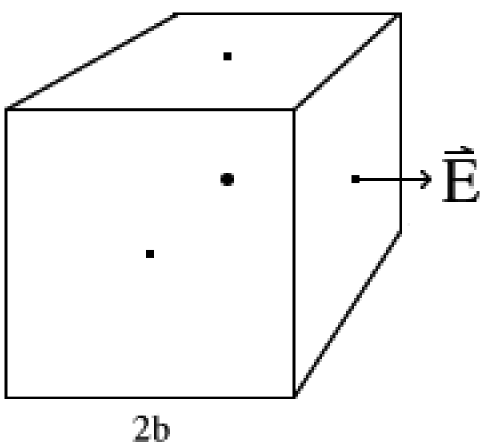
In this section, we will investigate the other type of vector integral that has come up in class, one in which the integrand is a dot product of two vectors. A good example is the flux calculation on the left hand side of Gauss' law.

$$\oiint_S \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

In Gauss' law, we calculate the total flux through a surface, so the integrand is a dot product of two vectors, the electric field and the surface element. Thus, the integrand (and the integral) is a scalar. However, to evaluate the integral for any particular case, you still have to come to grips with the vector nature of the parts of the integrand. Flux is defined as the integral overt a surface of the dot product of the electric field, ***E***, with the surface element, d***a***. Each surface element is a little vector. The magnitude is an element of area (think of it as a little disk or square) and the direction is perpendicular (normal) to the surface. That is,

$$d\vec{a} = \hat{n}da.$$

When we do the integral, we are totaling up the flux through each of the little surface elements.



We will consider a point charge of magnitude $Q = 5$ μC located at the center of a cube of side $2b = 15$ cm. First, calculate the correct total flux through the surface of the cube using Gauss' law, record the flux on the results page.

To calculate the flux through the cube directly, using the definition of flux rather than Gauss' law, is much harder. It can be done using Math261 techniques, but it is a relatively hard problem. We will approximate the flux in two different ways.

Method 1: The simplest (and worst) approximation is to break the surface of the cube up into six equal parts (each side becomes one part) and approximate the flux as

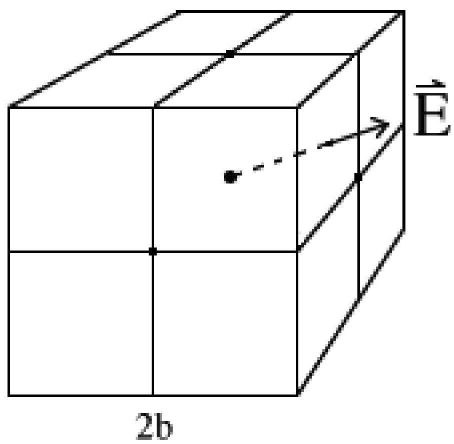
$$\varphi = 6\varphi_{side} = 6\left(\vec{E}_{center} \cdot \vec{A}\right)$$

The field at the center of the right side is ***E*** = $(1/4\pi\varepsilon0)(Q/b^2)\hat{\imath}$ and the area vector is ***A*** **=** $(2b)^2\hat{\imath}$. The field and the area are parallel ($\hat{\imath}\,\hat{\imath} = 1$) so the flux is

$$\varphi = 6\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{b^2}\left(4b^2\right) = \left(\frac{6}{\pi}\right)\frac{Q}{\varepsilon_0}$$

This is larger than the correct value by $(6/\pi) = 1.91$.

Method 2: divides the integral up into smaller pieces, but still takes advantage of the equal flux through each of the sides. The method is to divide each of the six sides into four squares, as shown in the figure. Now the total flux will be 24 times the flux through one of the small squares. Your job will be to calculate an approximate value for that flux, again using the field at the center, and the total area, in this case, you will also have to take the dot product carefully, since the field is no longer parallel to the surface normal ($\hat{n}$). Results page has spaces for several intermediate steps, and will help guide you through this calculation.



Name ____ **Date** _______________

Lab Partners ____

Calculus Workshop 1
Results

Please use the back of this page or attach another sheet if you need more room.

Section 1

1. What are the dimensions of your rubber rectangle?

   Length _____cm   Width ____cm   Mass _____g   #of pips _______

2. What is the surface number density for your rectangle?_____________________

3. If each pip carried a charge of 2μC, what would the surface charge density, σ_q, of your rectangle be?__________________________

Section 2

Summarize your derivation of the given result for the linear number density of your rectangle:

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1. What is the linear number density, λ_n for your cylinder? ______________________
2. What is the linear charge density λ_q using 2μC per pip? ______________________
3. Does the unrolled cylinder have the same linear charge density? ______________
4. What is the linear mass density of the cylinder? ______________________________
5. Is the linear mass density affected by rolling the cylinder tightly? _____________
6. How much would the room sized rubber mat weigh? __________________________
7. What would be the linear mass density of the room sized mat? ________________
8. Does the rectangular piece have a linear mass density in the perpendicular direction (along the short edge)? ____
9. What is the linear mass density in the short direction? _______________________

Overall

Explain the concept of linear charge density and surface charge density to “The Boss”.

What was good about this lab and what would you do to improve it?________________

Name ____ **Date** ________________

Lab Partners ____

Calculus Workshop 2
Results

Section 2

1. Set up the integral for the volume of the cone. ___________________________________

2. After evaluating this integral, what did you obtain for the volume of the cone?

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3. What are the units of the constant C in the formula $\rho = C(h - z)$? ______________

3. Set up the integral for the mass of the cone if it has a variable density $\rho = C(h - z)$:

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5. What did you obtain for the mass of the cone? _____________________________________

Section 3

1. Explain how to find the linear mass density of the rubber triangle. __________________

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2. What are the measurements of your triangle?

Length ______________ Height ______________ $\lambda_m(x)$_______________

3. Attach your excel graph

4. What is $\lambda_m(x)$ determined from your graph________________

5. What is your formula for the electric field at point P due to the linear charge density on the x axis?

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Overall

Explain to “The Boss” what integrals are used to calculate and why we need to use them. (He doesn’t want to hear that integrals are the area under a curve). ________

What was good about this lab and what would you do to improve it? _______________

Name ____ Date ______________

Lab Partners ____

Calculus Workshop 3
Results

Please use the back of this page or attach another sheet if you need more room.

Section 1

1. Using the exact formula, what is the electric field at point P?____________________________

2. Fill out the table below to approximate the value of the electric field at point P

Location of charge (y)	Amount of charge (dq)	Squared Distance to charge (r^2)	Magnitude of field ($dq/4\pi\varepsilon_0 r^2$)	Direction of field ($\hat{r}$)	Field of charge d**E**
4.5				( )i+( )j	$(1.86\text{x}10^3)$i+$(-1.05\text{x}10^3)$j
3.5				( )i+( )j	( )i+( )j
2.5				( )i+( )j	( )i+( )j
1.5				( )i+( )j	( )i+( )j
.5				( )i+( )j	( )i+( )j
–0.5				( )i+( )j	( )i+( )j
–1.5				( )i+( )j	( )i+( )j
–2.5				( )i+( )j	( )i+( )j
–3.5				( )i+( )j	( )i+( )j
–4.5				( )i+( )j	( )i+( )j
Total					( )**i**+( )**j**

3. Sketch a graph showing the charged distribution, the point P, dq, $\hat{r}$, and $d\mathbf{E}$ for the third row of the table (y=2.5) and the 8th row of the table (y=–2.5)

Flux using method 2

1. Calculate the exact flux outwards through the cube using Gauss' Law.___________________?

2. Distance from the point charge to the center of the square_____________________________?

3. Magnitude of the electric field at the center of the square_____________________________?

4. Direction ($\hat{r}$) of the electric field at the center of the square ( )**i**+( )**j** + ( )**k**?

5. Approximate flux (***E*** dot ***A***) _______________________________?

6. How does your value of flux compare to the exact value calculated using Gauss' Law? Explain to "The Boss" what makes this approximation so much better than the approximation in which the flux was estimate using whole sides instead of quarters.___________________________________

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Overall

What was good about this lab and what would you do to improve it?_______________

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