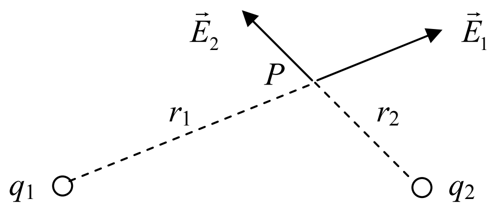
Calculus Workshop 2

Introduction

The electric field vector $\vec{E}_P$ of a point charge q at point P in space is given by the vector equation

$$\vec{E}_P = \frac{kq}{r^2}\hat{r}$$

where q is the charge in coulombs, k is the Coulomb constant 9.0×10^9 N-m^2/C^2, r is the distance between q and P, and $\hat{r}$ is a unit vector pointing from q to P. Of course, when q is negative, $\vec{E}$ points from P to q.

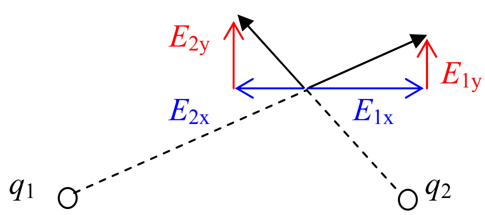


When two or more point charges are present, the electric field at point P is the sum of the field vectors due to each of the individual point charges (*the principle of superposition*):

$$\vec{E}_P = \Sigma\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots$$

As with any vector sum, it is often convenient to resolve each field vector into components and to sum the components separately:

$$\vec{E}_P = (\Sigma E_x)\hat{i} + (\Sigma E_y)\hat{j} + (\Sigma E_z)\hat{k}$$



The purpose of this workshop is to practice using the component method in evaluating sums of field vectors. Facility with this method will aid in the understanding of electric field integrals in Calculus Workshop 3.

LAB ASSIGNMENT:
Each student must submit Calculus Worksheets 2 & 3 before they leave the lab room. Your lab TA will help you with your calculations but will NOT do them for you!

Calculus Workshop 3

Introduction

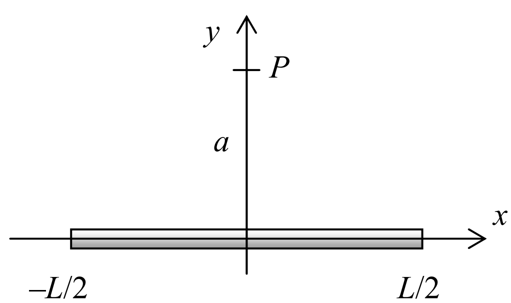
In Workshop 1 we calculated linear, surface, and volume densities for a variety of shapes. In Workshop 2 we found the electric field at a point in space due to a point charge configuration. Using the principle of superposition, we summed the field vectors due to each point charge using the component method. The fact that the charge configurations exhibited certain symmetries simplified our final results: the components perpendicular to the axis of symmetry cancelled out.

In this workshop we will be calculating the electric field at a point due to several different uniform charge distributions using the techniques developed in the previous workshops. The key difference, however, is the replacement of discrete vector sums by integrals:

$$\vec{E}_P = \int d\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

The general procedure is best illustrated by doing an example.

<u>Problem</u>: Find the electric field at point *P* due to a thin rod of uniform charge density as shown in the diagram. The rod has a total charge of *Q* and length *L* along the *x* axis centered at the origin. The coordinates of *P* are (0, *a*).

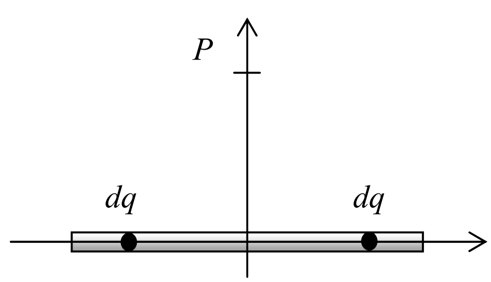


<u>STEP 1</u> – *Find the density of the rod* (λ *or* σ *or* ρ).

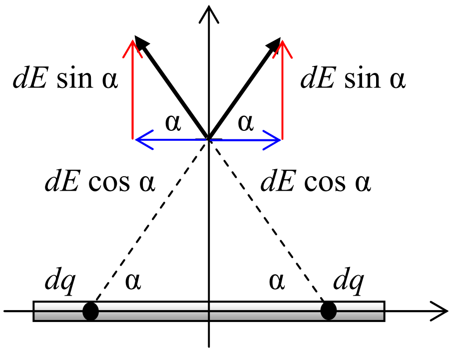
Since this is a uniform linear distribution, we must use $\lambda = \frac{charge}{length} = \frac{Q}{L}$.

<u>STEP 2</u> – *Identify the symmetry axis (if any). Pick two points along the charge distribution equidistant from and on opposite sides of the symmetry axis. Label each of them* "*dq*".

The symmetry axis is the *y* axis:



STEP 3 – *Draw the electric field at point P due to each of these infinitesimal point charges. Note that the magnitudes of these field vectors are equal but their directions are different. Resolve them into components and find the net field at P.*



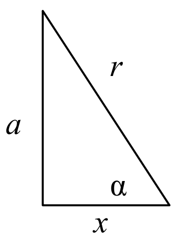
The x components cancel (due to symmetry) and the y components add, so the net field equals

$$d\vec{E} = (2dE\sin\alpha)\,\hat{j}$$

with

$$dE = \frac{k\,dq}{r^2}$$

STEP 4 – *Find expressions for all variables* (dq, r, α) *in terms of the coordinates of the charge distribution.*



$$dq = \lambda dx$$

$$r = \sqrt{x^2 + a^2} \qquad \sin\alpha = \frac{a}{r}$$

STEP 5 – *Integrate your net electric field over half of your charge distribution. (The factor of 2 has accounted for the other half.)*

$$\vec{E}_P = \int d\vec{E} = \int (2dE\sin\alpha)\,\hat{j} = \int_0^{L/2} 2\left(\frac{k\,dq}{r^2}\right)\sin\alpha\cdot\hat{j} = \int_0^{L/2} 2\left(\frac{k\,\lambda\,dx}{x^2+a^2}\right)\frac{a}{\sqrt{x^2+a^2}}\cdot\hat{j}$$

Remove all constants from the integral, simplify the integrand, and evaluate:

$$\vec{E}_P = \hat{j}(2k\lambda a)\int_0^{L/2}\frac{dx}{(x^2+a^2)^{3/2}} = \hat{j}(2k\lambda a)\left[\frac{x}{a^2\sqrt{x^2+a^2}}\right]_0^{L/2}$$ (Integral p.A-4 in text)

Substitute the limits of integration, replace $\lambda = Q/L$, and simplify to get

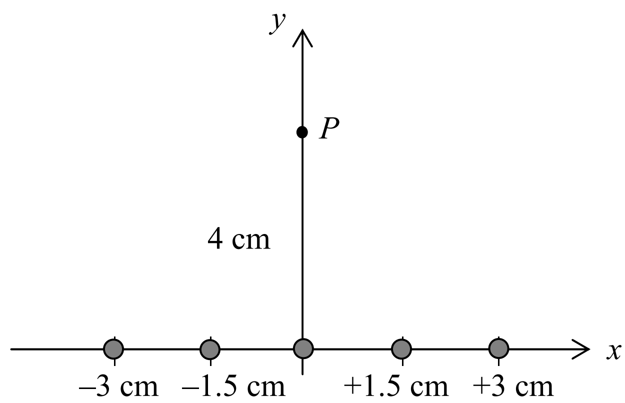
$$\vec{E}_P = \frac{kQ}{a\sqrt{(L/2)^2+a^2}}\,\hat{j}$$

Review these steps carefully – you are expected to understand them well enough to perform these types of integrals on your exams!

Calculus Worksheet 2

Name ____________________________________ **Date** ____________________

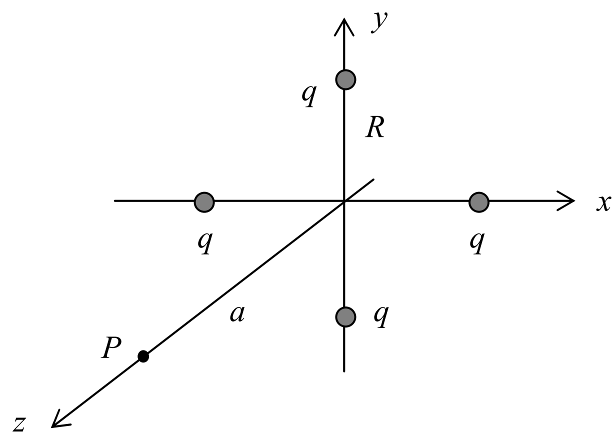
Problem 1



Five identical point charges $q = +10\ \mu\text{C}$ are situated along the x axis as shown.
(A) Calculate the electric field at point P located at coordinates (0, 4 cm). Express your answer in the form $\vec{E}_P = (\Sigma E_x)\hat{i} + (\Sigma E_y)\hat{j}$.

(B) The x component of your result should be identically zero. Why? Give a physical reason.

Problem 2



Four identical point charges q are a distance R from the origin in the xy plane as shown. (A) Calculate the electric field at point P located at coordinates $(0, 0, a)$. Express your answer in the form $\vec{E}_P = (\Sigma E_x)\hat{i} + (\Sigma E_y)\hat{j} + (\Sigma E_z)\hat{k}$.

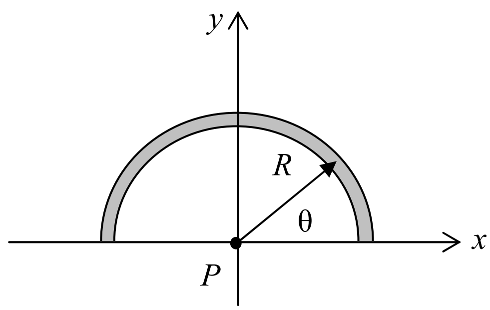
(B) Are there any components of your result that are identically zero? If so, explain why.

Calculus Worksheet 3

Name ______________________________ **Date** ________________

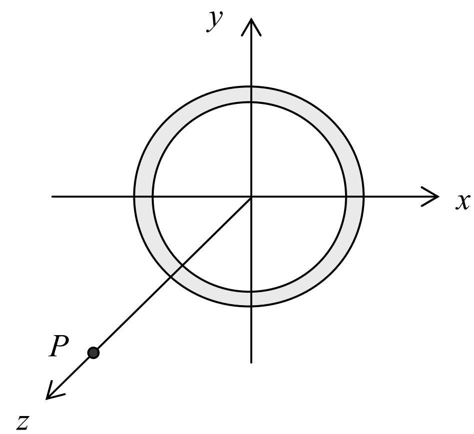
Directions: In each of the problems below, follow the steps illustrated in the introduction to find the electric field at point P due to the uniform charge distribution given.

Problem 1



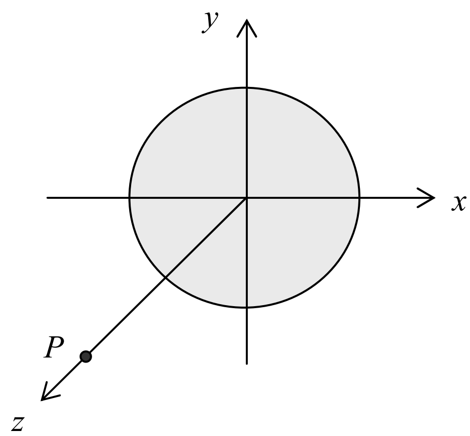
A thin wire shaped in a semicircle of radius R lies in the xy plane as shown in the diagram. Its total charge is Q. Point P is at the origin.

<u>Problem 2</u>



A thin wire shaped in a circular ring of radius R lies in the xy plane centered at the origin. Its total charge is Q. Point P has coordinates $(0, 0, a)$.

Problem 3



A thin circular disk of radius R lies in the xy plane centered at the origin. Its total charge is Q. Point P has coordinates $(0, 0, a)$.