Calculus Workshop 1

This is the first of a series of three labs exploring the important concept of charge density and its application to computing electric fields due to uniform charge distributions using the calculus of integration.

Introduction

Density, in its most general terms, expresses the concentration of a physical quantity, such as mass or electric charge, within a unit size, such as length, area, or volume. You have no doubt seen density defined as mass / volume, however this is one out of three possible definitions for density. The *average mass density* of an object may be defined as one of the following:

- *Linear* density (Greek letter *lambda*) $\lambda = \frac{mass}{length}$ SI unit: kg/m
- *Surface* density (Greek letter *sigma*) $\sigma = \frac{mass}{area}$ SI unit: kg/m^2
- *Volume* density (Greek letter *rho*) $\rho = \frac{mass}{volume}$ SI unit: kg/m^3

(The *average charge density* is obtained by replacing mass in kilograms with charge in coulombs in the above expressions) The type of density required in a physical situation depends on the shape of the object involved. For example, a wire is best described by a linear density; a tablecloth is best described by a surface density; a solid cube of cement is best described by a volume density; etc. An object is said to have a *uniform density* if its density is constant and equal to its average density.

In general, objects have densities that vary with position (called a *mass* or *charge distribution*). This implies that the linear, surface, or volume density is a function of position: $\lambda(x)$, $\sigma(x,y)$, $\rho(x,y,z)$. For example, suppose we have a solid rod, half of which is steel and half aluminum. Clearly, the mass density changes abruptly as you pass the midpoint of the rod. We say that the mass distribution is *nonuniform* because the density varies with position along the rod. If, on the other hand, the rod is completely steel, then its mass distribution is *uniform* because the density is uniform.

Given the density of an object, we can calculate its total mass m (or total charge q) according to the following chart:

Shape	Uniform Distribution	Nonuniform Distribution
Linear (L is total length)	m or $q = \lambda L$	m or $q = \int \lambda \, d\ell$
Surface (A is total area)	m or $q = \sigma A$	m or $q = \int \sigma \, dA$
Volume (V is total volume)	m or $q = \rho V$	m or $q = \int \rho \, dV$

Calculus Worksheet 1

Name ________________________________ **Date** ____________________

Problem 1 Given a rod of uniform density with a total mass of 1.5 kg and total length 45 cm, calculate
(A) the rod's linear density in kg/m.

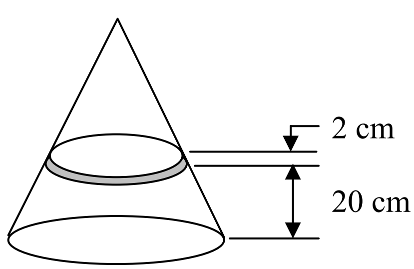
(B) the mass of a 3 cm section of the rod.

Problem 2 Given a circular disk of uniform density with a total charge of –60 μC and diameter 24 cm, calculate
(A) the disk's surface density in C/m^2.

(B) the charge on a washer (inner diameter = 9 cm, outer diameter = 18 cm) cut out of the disk.

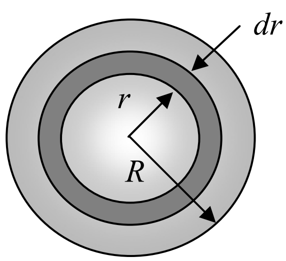
Problem 3 Given a right circular cone of uniform density of height 50 cm and base diameter 30 cm. If its total mass is 62.5 grams, calculate
(A) the cone's volume density in kg/m^3.

(B) the mass of a 2-cm slice of the cone parallel to and 20 cm above its base.



Problem 4 Given a straight wire of length L and charge density $\lambda(x) = \lambda_0\left(1-\frac{x}{L}\right)$, where λ_0 is a constant, calculate the total charge on the wire.

Problem 5 Given a circular disk of radius R and mass density $\sigma(r) = ar^2$, where a is a constant, calculate the total mass of the disk. (*Hint*: dA = area of washer = $2\pi r dr$.)



Problem 6 Given a solid sphere of radius R and charge density $\rho(r) = \frac{b}{r}$, where b is a constant, calculate the total charge in the sphere. (*Hint*: dV = volume of shell = $4\pi r^2 dr$.)