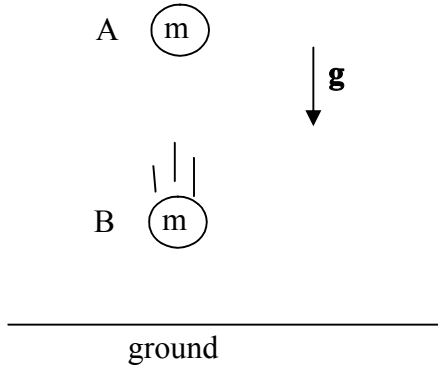


Electric Potential and Electric Potential Energy

To understand electric potential energy, an analogy with gravitational energy helps.

Gravitational Potential Energy: Imagine dropping a mass from rest at point A in a downward gravitational field that is uniform:



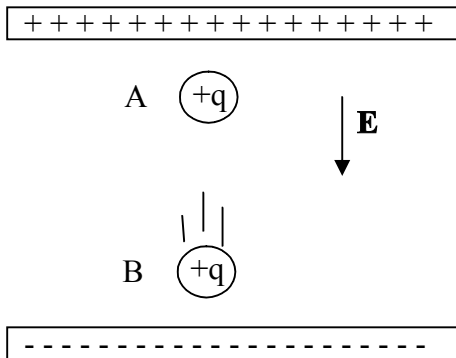
The gravitational force (\mathbf{F}_g) does work to transfer gravitational potential energy (GPE) into kinetic energy (KE). The loss in GPE equals the gain in KE

Work done by grav. field = ΔKE

If no other force act, then according to conservation of energy:

$$KE_A + GPE_A = KE_B + GPE_B$$

Electrical Potential Energy: Now, imagine releasing a positive charge in a uniform downward electric field between two capacitor plates:



The electric force force (\mathbf{F}_e) does work to transfer electric potential energy (EPE) into kinetic energy (KE). The loss in EPE equals the gain in KE

Work done by elec. field = ΔKE

If no other force act, then according to conservation of energy:

$$KE_A + EPE_A = KE_B + EPE_B$$

Questions: What would happen negative charge released at point B in the electric field above?

For the negative charge released from rest at B, would it gain or lose EPE? Explain.

ELECTRIC POTENTIAL (V): Electric Potential is the ratio of EPE that a test charge has at a point in space to the value of that test charge.

$$V = \frac{EPE}{q_o} \quad \text{units} = J/C = \text{volt}(V)$$

Because it's defined in terms of energy, electric potential is a scalar quantity. However, it may be positive or negative. In the formula, the sign of the test charge is important. Electric potential is an abstract, but very useful, quantity.

Example: The electric potential at a point in space is 9 volts. What is the electric potential energy of a proton placed at that point?

$$EPE = Vq_o = (12 \frac{J}{C})$$

Question: The electric potential at a point in space is 9 volts. What is the electric potential energy of an electron placed at that point?

Imagine a charge accelerating between two points in space, like points A & B discussed earlier. Using the conservation of energy

$$EPE_A + KE_A = EPE_B + KE_B$$

$$EPE_A - EPE_B = KE_B - KE_A$$

$$q_o V_A - q_o V_B = \Delta KE$$

$$q_o (V_A - V_B) = \Delta KE$$

$$-q_o (V_B - V_A) = \Delta KE$$

$$-q_o \Delta V = \Delta KE$$

Using the work-energy theorem:

$$W_{\vec{E}} = \Delta KE = -q_o \Delta V$$

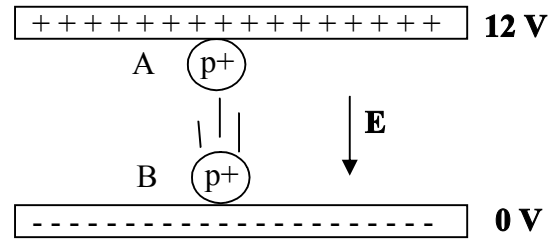
The equation above represents the work done by the electric force in accelerating a charge between two points. If an external agent controls the motion of a charge between two points in an electric field, it will work against the electric field. The sign will be opposite as follows:

$$W_{\text{external force}} = -W_{\vec{E}} = -\Delta KE = q_o \Delta V$$

ΔV is called the **Electric Potential Difference** (units: J/C=V) and it represents the Joules of electric potential energy transferred to or away from a Coulomb of charge passing between two points.



Example: A proton is released from rest at point A in the capacitor shown to the right. Plate A is at a potential of 12 V and plate B is “grounded” with an assumed potential of 0 V. How fast will the proton be moving when it arrives at B?



$$\begin{aligned}
 EPE_A + KE_A &= EPE_B + KE_B \\
 q_{p^+}V_A + KE_A &= q_oV_B + KE_B \\
 q_{p^+}V_A + 0 &= 0 + \frac{1}{2}m_{p^+}v^2 \\
 v &= \sqrt{\frac{2q_{p^+}V_A}{m_{p^+}}} = \sqrt{\frac{2(1.603 \times 10^{-19} C)(12 J/C)}{1.67 \times 10^{-27} kg}} = 4.8 \times 10^4 m/s
 \end{aligned}$$

Question: How much work is done by the electric field as it accelerates the proton?

Question: How much work is done by an external agent to move an electron constantly from A to B?

The Electron-volt (eV) is another unit for energy that is convenient to use when dealing with fundamental charges. It is defined as the kinetic energy that an electron gains as it accelerates through a potential difference of 1 Volt.

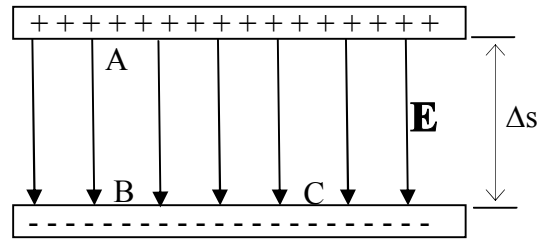
$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

In the previous question, the work would be:

$$W_{AB} = 1.92 \times 10^{-18} \text{ J} \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) = 12.0 \text{ eV}$$

The relationship between the Electric Field and a changing potential

The work done moving a positive charge from Point A to point B in the capacitor shown can be calculated in two ways:



$$W_{\vec{E}} = \vec{F}_e \cdot \vec{\Delta s} = F_e \Delta s \cos(0^\circ) = (F_e) \Delta s = (q_o E) \Delta s$$

and

$$W_{\vec{E}} = -q_o \Delta V$$

Equating these two expressions:

$$q_o E \Delta s = -q_o \Delta V$$

$$E = -\frac{\Delta V}{\Delta s} \quad \text{unit} = \frac{V}{m}$$

This expression for the electric field is known as the “potential gradient.” The greater the change in electric potential per meter, the greater the strength of the electric field.

Question: Show that a V/m = N/C.

How much work would it take to move the charge from B to C? This can be shown to be zero in two ways:

$$W_{\vec{E}} = \vec{F}_e \cdot \vec{\Delta s} = F_e \Delta s \cos(90^\circ) = 0 \quad \text{and} \quad W_{\vec{E}} = -q_o \Delta V = -q_o(0) = 0$$

The total work from A to C may be found as follows:

$$W_{AC} = W_{AB} + W_{BC} = W_{AB} + 0 = W_{AB}$$

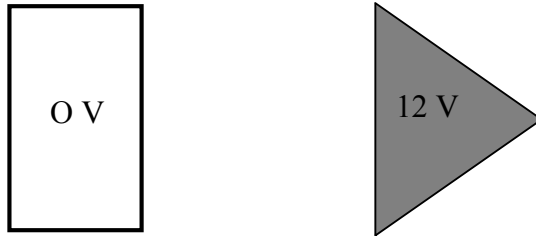
The total work done by the electric field is independent of path because the electric force is a conservative force.

Question: The potential difference between two capacitor plates is 12V and they are spaced 2.0mm apart. What is the magnitude of the electric field between the plates?



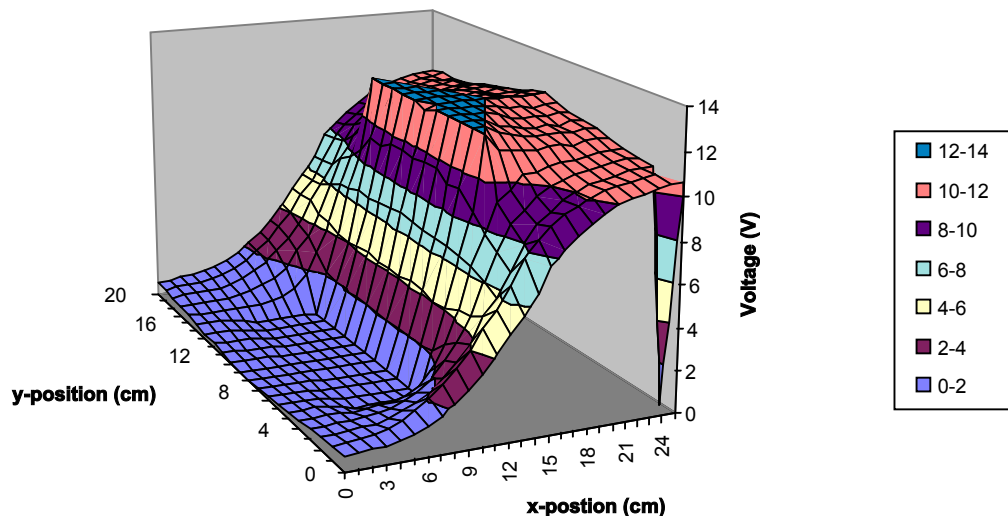
Visualizing Electric Potential & Energy

Imagine two conductors, one hollow and one solid. The hollow conductor on the left is grounded (0V) and the conductor on the right is at 12 V.



The potential increases from the left conductor to the right conductor. If we measure the potentials at many points between the conductors, we can plot the potentials in a 3-dimensional graph as shown:

Electric Potential Landscape



Where would you expect the electric field to be the greatest magnitude? According to the potential gradient formula, it would occur where $\Delta V/\Delta s$ is the greatest which is where the graph's slope is the steepest. This appears to be at the front 2 corners of the triangular conductor (sharp points...this explains lightning rods).

Questions:

Does the electric field point uphill or downhill? Explain.

Which way would a proton accelerate if placed in between the conductors?

Which way would an electron accelerate if placed in between the conductors?

Would the electron or the proton have a greater magnitude of acceleration? Explain.



The Electric potential in the vicinity of point charges:

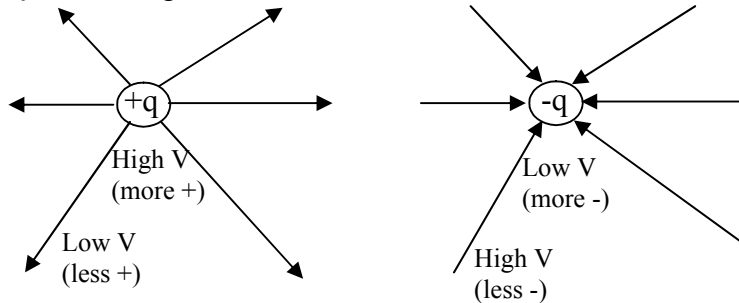
In the region around a point charge the electric potential is:

- 1) directly related to q (the value of the charge, sign is important)
- 2) inversely related to r (the distance from the point charge)

It can be derived using calculus that:

$$V = \frac{kq}{r} \quad \text{where } V = 0 \text{ at } r = \infty$$

The potential is positive close to positive point charges and negative close to negative point charges



- Positive test charges placed in an electric field will ALWAYS tend to accelerate from regions of high potential to regions of low potential (i.e., they accelerate in the direction of the electric field).
- Negative test charges placed in an electric field will ALWAYS tend to accelerate from regions of low potential to regions of high potential. (i.e., they accelerate opposite to the direction of the electric field).

Example: a) Determine the electric potential at a distance of 3.0m from a $-18\mu\text{C}$ point charge.

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2})(-18 \times 10^{-6} \text{C})}{3.0\text{m}} = -5.4 \times 10^4 \text{V}$$

Notice that the denominator is NOT squared like it is for force and fields.

b) What is the electric potential energy of a $+2.0\mu\text{C}$ charge placed at the same point?

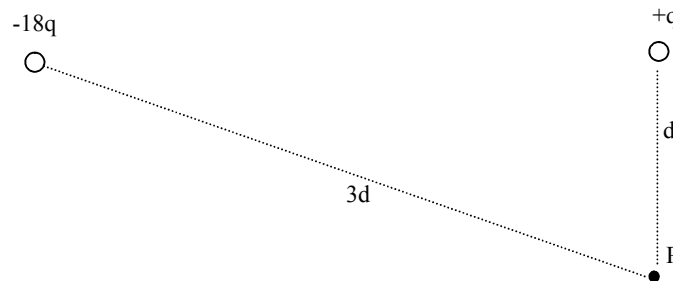
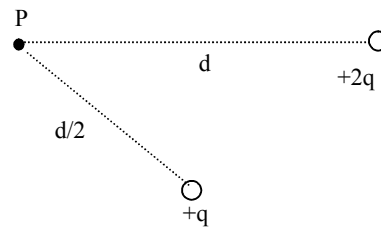
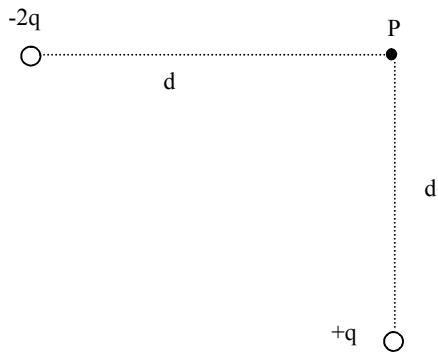
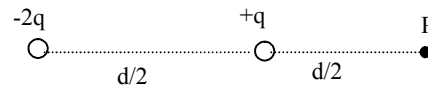
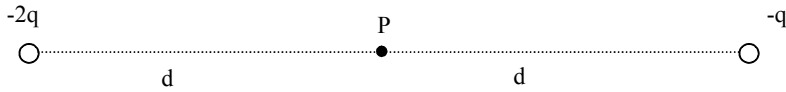
$$EPE = q_o V = (+2.0 \times 10^{-6} \text{C})(-5.4 \times 10^4 \text{J/C}) = -0.11 \text{J}$$

A negative EPE means that, relative to a zero EPE infinitely far away from the negative point charge, the $+2.0\mu\text{C}$ charge is at a lower EPE. It will tend to accelerate toward the negative point charge to even lower potentials.

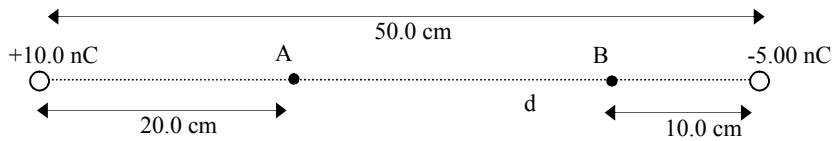
Question: Determine the sign of the EPE for a negative test charge placed at the same position. Explain the sign.

Electric Potential from multiple point charges

Write an expression for the total electric potential at point P in each figure



Total Electric Potential from multiple point charges Numerical Example



Calculate the total electric potential at point A in above.

The total potential is the scalar sum (with the proper sign) of the potential from each neighboring point charge:

$$V_{Total} = V_{A_{+10}} + V_{A_{-5}} = \frac{kq_{+10}}{r_{+10}} + \frac{kq_{-5}}{r_{-5}}$$

$$V_{Total} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(10.0 \times 10^{-9} C)}{0.200m} + \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(-5.00 \times 10^{-9} C)}{0.300m}$$

$$V_{Total} = 449.5V - 149.8V = 300.V$$

Question #1: Show that the total potential at point B is -225V.

Question #2: If an electron is released from rest at point B (-225 V), show that its speed reaches 1.36×10^7 m/s when it reaches point A (300 V).

Question #3: Show that the average electric field between A and B is 2600N/C

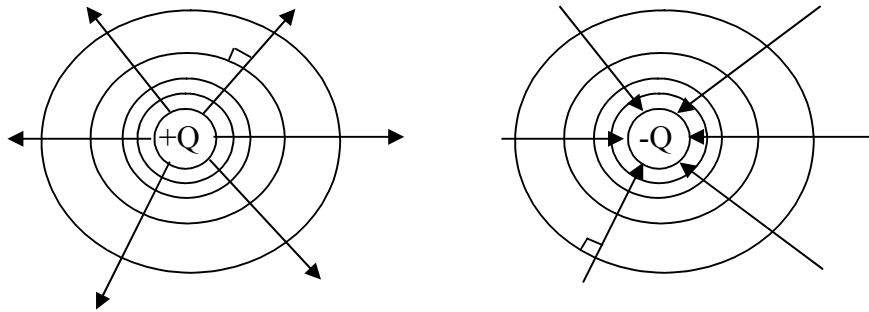
Fill out the following table with the correct signs (+ or -):

	ΔV	W_E	W_{ext}	ΔKE	ΔEPE
Proton going from high V to low V					
Proton going from low V to high V					
Electron going from high V to low V					
Electron going from low V to high V					



Equipotential Lines & Surfaces

Regions of constant potential



- The equipotential lines for the two point charges are concentric circles.
- The equipotential lines have no direction and don't have arrows.
- The equipotential lines must be perpendicular to the electric field lines ($E = \Delta V / \Delta s = 0$ because $\Delta V = 0$ along an equipotential by definition, so there is no component of the electric field in the direction on the equipotentials).
- No work is done by the electric field when charges are moved along an equipotential ($W_E = -q_0 \Delta V = 0$), just like the gravitational field does no work when a hiker walks along contour lines indicated on a map ($W_g = mgh = 0$).

Conductive surfaces in Equilibrium:

- The electric field must be perpendicular to the surface, otherwise free electrons would be accelerating parallel along the surface and it wouldn't be in equilibrium. This implies that the surface of the conductor is an equipotential.
- Electric shielding occurs for all conductors at equilibrium, therefore the electric field must be ZERO inside the conductor. *This implies that the entire conductor is an equipotential at equilibrium!* All points on and inside the conductor are at constant potential.

Look back the "Visualizing Electric Potential and Energy" example. Here is the corresponding equipotential diagram. Answer the questions below:

How much work would it take to move an electron up the 12cm line between the plates? How about within the conductors?

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

How can you visualize where the electric field is the greatest on the diagram?

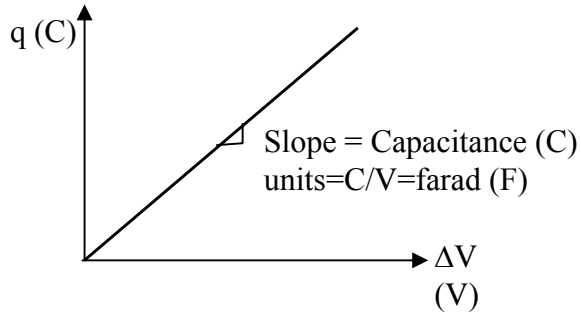
What can you say about the electric field within the hollow rectangular conductor?
How about the electric potential?

What can you say about the electric field within the solid triangular conductor on the right? How about the electric potential?



Capacitors

Capacitors consist of two parallel metal plates separated by an insulator. A power supply moves electrons from one plate to another such that one plate obtains a charge of $+q$ and the other $-q$. The bigger the potential difference between the plates, the more charge that is separated.



Equation for line:

$$y = mx + b$$

$$q = C\Delta V + 0$$

$$q = C\Delta V$$

The capacitance (slope) remains constant for a given capacitor, so changes in the potential difference and charge will NOT affect the capacitance (i.e., the charge to voltage ratio stays the same). The only way to change the capacitance is to alter the geometry and the materials within the capacitor.

Capacitance depends--

- 1) directly on the area of each plate (A) (but not both plates added together)
- 2) inversely on the spacing between the plates (d)
- 3) directly on the dielectric constant (represented by κ). This is a unitless quantity that represents the type of insulator (dielectric) between the plates. The insulator gets polarized and allows more charge to be stored on the plates with less ΔV .

Some representative κ values are:

vacuum	$\kappa=1.00000$
air	$\kappa=1.00054$
paper	$\kappa=3.3$

$$C = \frac{\epsilon_0 \kappa A}{d} \quad \text{where } \epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

ϵ_0 is called the “permittivity of free space” and is related to the electric constant k (don’t get this mixed up with the dielectric constant κ) as follows:

$$k = \frac{1}{4\pi\epsilon_0}$$

Questions: What happens to the capacitance of a capacitor when—

- a) its plate area is tripled?
- b) the distance between the plates is doubled?
- c) the charge on the plates is doubled?
- d) a vacuum between the plates is replaced with paper (keeping the distance the same)?



Capacitor Example: A capacitor stores $5.3\mu\text{C}$ of charge when connected to a 6.0-V battery. The plates are circles with a radius of 0.78m and they are separated by paper.

a) How much charge does it store when connected to a 9.0-V battery?

$$C = \frac{q_6}{\Delta V} = \frac{5.3 \times 10^{-6} \text{ C}}{6.0 \text{ V}} = 88.33 \mu\text{F}$$

$$q_9 = C\Delta V = (88.33 \mu\text{F})(9.0 \text{ V}) = 8.0 \mu\text{C}$$

Notice that the capacitance stayed the same, regardless of charge or potential difference.

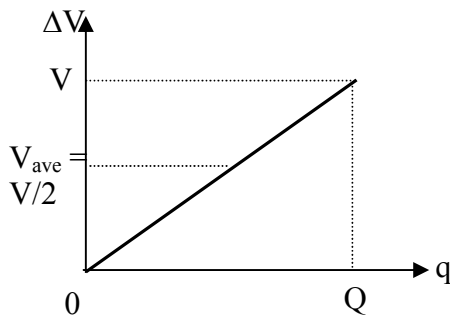
b) What is the distance between the circular plates?

$$C = \frac{\epsilon_0 \kappa A}{d} \quad \text{so} \quad d = \frac{\epsilon_0 \kappa A}{C} = \frac{\epsilon_0 \kappa (\pi r^2)}{C}$$

$$d = \frac{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2})(3.3) \{ \pi (0.78)^2 \text{ m}^2 \}}{8.0 \times 10^{-6} \text{ F}} = 7.0 \mu\text{m}$$

Energy Storage in Capacitors

It takes work to charge a capacitor, and, as a result, energy is stored in the capacitor's electric field. To charge an uncharged capacitor to a final charge Q , a potential difference must be applied across the plates. As the amount of charge increases, the potential difference across the plates increases as shown.



Since the potential difference builds up linearly with charge, we must use the average V to find the amount of work needed to charge the capacitor:

$$W = Q(V_{\text{ave}}) = Q(V/2)$$

Since $Q = CV$,

$$E_{\text{capac}} = 1/2 CV^2 = Q^2/2C$$

Question: A capacitor stores 5J of energy when connected to a potential difference of 12 Volts. How much energy does it store when connected across a 24V potential difference?

Energy Density in Capacitors

Energy density for a capacitor is a measure of the energy in its electric field per volume of space. For plate distance d and area A , the volume within the capacitor is $\text{Volume} = Ad$

$$E_{\text{capac}} = \frac{1}{2} C(V)^2 = \frac{1}{2} (C)(Ed)^2 = \frac{1}{2} \left(\frac{\epsilon_0 \kappa A}{d} \right) (Ed)^2 = \frac{1}{2} \epsilon_0 \kappa (Ad) E^2 = \frac{1}{2} \epsilon_0 \kappa (\text{Volume}) E^2$$

$$\frac{E_{\text{capac}}}{\text{Volume}} = \frac{1}{2} \epsilon_0 \kappa E^2$$

