Electrostatics
...the study of electrical charges at rest on objects.

A very basic model of the atom helps explain electrostatics:

![Diagram of electron, proton, and neutron]

Important features:
1) The center of the atom is a tiny concentrated region of positive particles (protons) and neutral particles (neutrons) called the nucleus. The number of protons in the nucleus uniquely determines what element it is. This number will not change unless a “drastic” nuclear event occurs (i.e. fission or fusion).
2) In the large region of space around the tiny nucleus there is likelihood that you will find the negative particles (electrons).
3) In neutral atoms, the number of electrons equals the number of protons.

Electrons are the “mobile fluid” responsible for charging solid objects. Therefore, to charge (originally neutral) atoms & solid objects:
1) They must **gain extra electrons** to obtain a **negative** electric charge.
2) They must **lose electrons** to obtain a **positive** charge.

**NOTE:** A solid object will always have the same amount of protons before and after gaining an electrostatic charge.

**Law of Conservation of Charge**: Electric charge can NEVER be created or destroyed but may be transferred (moved) between objects.

Important quantities to have available for every test & quiz this semester:
- Mass of proton: \(1.673 \times 10^{-27}\) kg
- Mass of neutron: \(1.675 \times 10^{-27}\) kg
- Mass of electron: \(9.109 \times 10^{-31}\) kg

**NOTE:** The electron is approximately 2000X less massive than protons. This also helps to explain why electrons are the mobile particles in electrostatics.

In 1910, Milliken found in his famous oil drop experiment that atomic charges always came in multiples of an elemental charge (e):

\[ e = 1.601 \times 10^{-19}\text{ C} \quad \text{where } C = \text{Coulomb, the SI unit of charge} \]
The particles of the atom have the following charge (q):

\[ q_{\text{proton}} = +e \quad q_{\text{electron}} = -e \quad q_{\text{neutron}} = 0 \]

Example: How many protons does it take to accumulate 1.00-C of positive charge?

\[ N = \frac{q}{e} = \frac{1.00 \text{C}}{1.601 \times 10^{-19} \text{C}/\text{proton}} = 6.26 \times 10^{18} \text{ protons} \]

Try this example: Two identical, isolated metal spheres are given charges. Sphere #1 has a charge of +15.0 \( \mu \)C and Sphere #2 has a charge of -5.0 \( \mu \)C. They are touched and then separated. Did Sphere #2 gain or lose electrons?

How many electrons transferred?

Show that Sphere #2 lost a total of 5.7 \( \times 10^{-17} \) kg of mass.

Basic observations in electrostatics:

1) Objects charged in the same way repel
2) Objects charged oppositely attract
3) Neutral objects are attracted to both positive and negative objects.

The electric forces \( F_e \) are like any other forces and must obey Newton’s laws:

1) Objects will maintain constant velocity unless the forces are unbalanced.
2) Acceleration is in the direction of the net force, is directly proportional to net force, and inversely proportional to the mass of the object:
   \[ a = \frac{F_{\text{net}}}{m} \]
3) Forces are interactions between two objects. Forces are ALWAYS equal in magnitude and opposite in directions (regardless of the properties of the individual objects).
**Conductors** are materials in which electrons flow easily from atom to atom.

**Insulators** are materials in which electrons are tightly bound to a particular atom.

The conductor/insulator continuum for different materials:

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**Why are neutral conductors and neutral insulators attracted to charge objects?** This can be explained with the concepts of electric induction and polarization:

**Electric Induction:** When a charged object is brought near a neutral conductor, the electrons are induced to move to one side of the conductor (remember: protons stay fixed in solids!). Note how the electrons are repelled to the right hand side of the conductor shown below, which leaves the protons on the left hand side.

![Electric Induction](image)

**Electric Induction**

Draw the corresponding picture if the object is positive:

**Electric polarization:** When a charged object is brought near an insulator, the electrons will not flow to the one side of the entire object. Rather, the electrons tend move within the atom such that the atoms are polarized.

![Electric Polarization](image)

**Electric Polarization**
Draw the corresponding picture for electric polarization if the object is positive:

There are three common ways objects can pick up a net charge:

1) **Friction**: For example, when wool is rubbed on plastic, the plastic gains electrons to become negative and the wool loses electrons to become positive.

   Question: Silk is rubbed on glass, leaving the glass with a positive charge. **Explain** what happens in terms of electrons & protons:

2) **Contact**: When a charged object touches a neutral object, the neutral object picks up the same charge.

3) **Charging by induction and grounding**: When a charged object is brought near a grounded conductor, the conductor picks up an opposite charge when the grounding wire is removed.

   ![Charging by Induction and grounding](image)

   Electrons from ground neutralize right end of the conductor giving the conductor a net negative charge.

Draw the corresponding picture if the original object is negative:

Answer the following question and check your answer with your neighbors:

In an electrostatics experiment, two pieces of tape attract each other. You can **firmly** conclude that:

A) at least one of the pieces of tape is charged.
B) one of the pieces of tape is neutral.
C) the two pieces of tape are oppositely charged.
D) you did the experiment incorrectly.
Coulomb’s Law

Used to determine electric forces between point charges

The magnitude (size) of the electric force between two charged objects depends-

1) Directly on the strength of the charge on object #1 \(q_1\)
2) Directly on the strength of the charge on object #2 \(q_2\)
3) Inversely on the square of the distance between the charges \(r\)

\[
|F_e| = \frac{kq_1q_2}{r^2}
\]

where \(k = 8.99 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}\)

The direction of the force is along the line connecting the two charges. Use the fact that like charges repel and opposites attract to determine the direction.

Example: Calculate the electric force between a +2.0 C charge and a -3.0 C charge.

\[
|F_e| = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \frac{Nm^2}{C^2})(2.0C)(3.0C)}{(2.0m)^2}
\]

\[= 1.4 \times 10^{10} \text{ N attraction}\]

This force is enormous! It is nearly impossible to isolate charges of this magnitude because of the forces involved.
Comparing forces between point charges: The figure below shows two positive charges repelled by an electric force, \( F_e \). Show how the force will change in each of the cases below with arrows of the correct length and direction.
Determining net forces on a point charge from neighboring point charges:
Show all the electric forces on the SYSTEM that is identified by the dashed circle and also show the direction of the NET electric force using vector techniques.
Example of using Coulomb’s Law to find the net force on a point charge:
Find the net force on the -2.0\(\mu\)C charge in the figure below:

Step 1: Draw a vector diagram representing the force on the charge of interest.

```
\begin{align*}
\vec{F}_{-3} &= (8.99 \times 10^9 \text{Nm}^2/C^2)(2.0 \times 10^{-6}\text{C})(3.0 \times 10^{-6}\text{C}) \\
&= \frac{(0.4m)^2}{(8.99 \times 10^9 \text{Nm}^2/C^2)(2.0 \times 10^{-6}\text{C})(3.0 \times 10^{-6}\text{C})} = 0.337\text{N up}
\end{align*}
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\begin{align*}
\vec{F}_{+10} &= (8.99 \times 10^9 \text{Nm}^2/C^2)(2.0 \times 10^{-6}\text{C})(10.0 \times 10^{-6}\text{C}) \\
&= \frac{(0.5m)^2}{(8.99 \times 10^9 \text{Nm}^2/C^2)(2.0 \times 10^{-6}\text{C})(10.0 \times 10^{-6}\text{C})} = 0.719\text{N down and to the left}
\end{align*}
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Step 3: Determine the x- & y-components of each force, being careful with the signs

<table>
<thead>
<tr>
<th>Force</th>
<th>x-component</th>
<th>y-component</th>
</tr>
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<tbody>
<tr>
<td>(\vec{F}_{-3})</td>
<td>0</td>
<td>+0.337N</td>
</tr>
<tr>
<td>(\vec{F}_{+10})</td>
<td>(-0.719\cos\theta = -0.719 \times \cos(30) = -0.432)</td>
<td>(-0.719\sin\theta = -0.719 \times \sin(30) = -0.575)</td>
</tr>
<tr>
<td>(\vec{F}_{\text{net}})</td>
<td>-0.432N</td>
<td>-0.238N</td>
</tr>
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</table>

Step 4: Sketch the net force and determine its magnitude and direction:

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\begin{align*}
|\vec{F}_{\text{net}}| &= \sqrt{F_{\text{net}_x}^2 + F_{\text{net}_y}^2} = 0.493\text{N} \\
\gamma &= \tan^{-1}(\frac{F_{\text{net}_y}}{F_{\text{net}_x}}) = 29^\circ \text{ as shown (209^\circ from +x axis)}
\end{align*}
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Practice Problem: Show that the net force on the +10\(\mu\)C charge is 3.48N at 9.5 degrees from the +x-axis.
Electric Fields

In the vicinity of charged objects, there exists an electric field in which other charged objects will experience forces.

The magnitude of the electric field at a point in space is defined as the ratio between the electric force on a positive test charge \( (q_o) \) and the value of that test charge:

\[
|E| = \frac{F_e}{q_o} \quad \text{Units} = N/C
\]

The electric field, like the electric force, is a vector quantity and requires direction. The direction of the electric field is determined by the direction of the electric force on a positive test charge. Thus, positive charges placed in an electric field experience electric forces in the direction of the field, whereas negative charges experience electric forces opposite to the direction of the field.

Example: At the point P shown, there is an electric field of 12 N/C to the right. What is the electric force on a proton placed a point P?

\[
|F_e| = EQ_o = (12N/C)(1.603 \times 10^{-19} C) = 1.9 \times 10^{-18} N
\]

the direction is to the right, in the direction of the electric field, because the proton is positive, just like the test charge that defines electric fields.

Question: What is the electric force on an electron placed a point P? (Remember, for all vector quantities, to specify magnitude and direction!)

Which particle (the electron or proton) placed at point P would accelerate at a greater rate? Explain.
**Electric field for point charges:** The electric field at a distance $r$ from a central point charge $q$ is determined from Coulomb's Law:

$$|E| = \frac{F_e}{q_o} = \frac{kq}{r^2} = \frac{kq}{r^2}$$

Notice how the electric field at point in space does NOT depend on the test charge placed there, but does depend on the value and position of neighboring charges.

A classic electric field problem is determining the value of the net electric field at a point in space from neighboring point charges. We will practice our vector techniques by sketching the net electric field at point $P$ in each case below:
Example of finding the net electric field due to neighboring point charges:
Find the net electric field and point P in the figure below:

Step 1: Draw a vector diagram representing the electric field on a positive (always!) test charge placed at P

Step 2: Determine the magnitude of each individual field

\[ |E_{-3}| = \frac{(8.99 \times 10^9 \text{Nm}^2/C^2)(3.0 \times 10^{-6} \text{C})}{(0.4 \text{m})^2} = 1.69 \times 10^5 \text{N/C down} \]

\[ |E_{+10}| = \frac{(8.99 \times 10^9 \text{Nm}^2/C^2)(10.0 \times 10^{-6} \text{C})}{(0.5 \text{m})^2} = 3.60 \times 10^5 \text{N/C up and to the right} \]

Step 3: Determine the x- & y-components of each field, being careful with the signs

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<tr>
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<tr>
<td>( E_{+10} )</td>
<td>+3.60 \times 10^5 \cos \theta = \frac{3.60 \times 10^5 \text{N/C}}{2} \times \frac{3}{5} = 2.16 \times 10^5 \text{N/C}</td>
<td>+3.60 \times 10^5 \sin \theta = \frac{3.60 \times 10^5 \text{N/C}}{2} \times \frac{4}{5} = 2.88 \times 10^5 \text{N/C}</td>
</tr>
<tr>
<td>( E_{\text{net}} )</td>
<td>+2.16 \times 10^5 \text{N/C}</td>
<td>+1.19 \times 10^5 \text{N/C}</td>
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Step 4: Sketch the net field and determine its magnitude and direction:

\[ |E_{\text{net}}| = \sqrt{E_{\text{net},x}^2 + E_{\text{net},y}^2} = 2.47 \times 10^5 \text{N/C} \quad \gamma = \tan^{-1}\left(\frac{E_{\text{net},y}}{E_{\text{net},x}}\right) = 29^\circ \text{as shown} \]

Practice Problem: Show that the acceleration of an electron placed at point P is 4.35 \times 10^{16} \text{m/s/s @} 209 \text{ deg.}
Electric Field Lines

Electric fields are sometimes called “lines of force” or “flux lines” because the show the direction of forces on positive test charges placed near a central charge. Electric field lines can be visualized around charges. For example, the first diagram below shows that a positive test charge is repelled from a positive central charge (+Q). The second diagram shows that the same positive test charge is attracted to a negative central charge (-Q).

When drawing electric field line patterns, there are some basic facts to keep in mind:

1) Electric field lines always point away from positive charges and toward negative charges.
2) The number of field lines entering or leaving a charge is proportional to the magnitude of each charge.
3) Field lines never cross.
4) The electric field is stronger in regions where the field lines are closer together and weaker in regions where the lines are far apart.

For example, what would the electric field pattern look like if two protons (+2e) were in the vicinity an electron (-e)? The following diagram will show the pattern:

Notice how twice as many field lines leave the positive charge as enter the negative charge.
What do field lines look like in the vicinity of two repelling charges? For example, the following diagram shows the electric field pattern in the vicinity of a +6 Coulomb charge and a +4-Coulomb charge:

Notice how the field lines never cross and how more field lines leave the +6C charge than leave the +4C charge. Also, notice the empty region of space between the charges. This indicates that the electric field is weak in the region between the charges. Use a vector diagram to show that there is a point nearer to the +4C charge where the net electric field is zero. How could you calculate the exact location if the objects are a known distance, d, apart?

Now try sketching the electric field line pattern in the vicinity of a -50C and a -100C charge.

**Parallel plate capacitor**: The electric field for two parallel plates, each with equal and opposite charge is shown to the right. This device is produces a nearly uniform electric field in the central region between the plates.
Electric Shielding

When a conductor is in its stable state (called equilibrium), excess electric charges on a conductor will reside on the outer surface such that the electric field in the entire central region of the conductor is zero. This is true regardless of whether the conductor is hollow or solid.

For a negative conductor, this electric shielding occurs because the free electrons repel to the outside surface, making the inside neutral:

For a positive conductor, this electric shielding occurs because the free electrons neutralize the inside of the conductor, leaving positive charge (unpaired protons) on the surface:

Examples of Electric Shielding:

1) Safety in cars & airplanes truck by lightning
2) Metal cases for sensitive electronic equipment

Additional electrostatics topics:

1) Lightning & Lightning rods
2) Van de Graaf generator