## Simple Harmonic Motion I

## Objectives

In this lab you will

- test the theory of simple harmonic motion in the case of a simple pendulum and a Hooke's law spring.
- measure the position and velocity using the Vernier Motion Detector.
- plot your data and analyze it using the Vernier Logger Pro ${ }^{\mathrm{TM}}$ software.


## Equipment

Vernier Motion Detector (with clamp), Vernier LabPro ${ }^{\text {TM }}$ system (includes computer and Logger Pro ${ }^{\text {TM }}$ ), metal ball, wooden ball, support bracket (clamped to a small stand), a set of slotted masses, mass hanger, tape, string, rubber bands, and a meterstick.

## Theory

Systems undergoing simple harmonic motion share the following general characteristics:

- Small amplitude displacements from equilibrium.
- A restoring force proportional to the displacement.
- Periodic motion described by sines and cosines.

For a simple pendulum of length $\ell$, the period $T$ is given by

$$
T=2 \pi \sqrt{\frac{\ell}{g}}
$$

Equation 1

For a mass attached to a Hooke's law spring of constant $k$, the period given by

$$
T=2 \pi \sqrt{\frac{m}{k}}
$$

## Equation 2

Suppose that the system (pendulum or spring) is initially displaced by magnitude $A$. Its displacement $s(t)$ for all times $t \geq 0$ is given by the function

$$
s(t)=A \cos (\omega t)
$$

Equation 3
where $\omega=2 \pi / T$ is the angular frequency of the oscillation (in rad/s). From Equation 3 one can derive the instantaneous velocity $v(t)$ and acceleration $a(t)$ functions (using elementary calculus):

$$
\begin{array}{ll}
v(t)=-\omega A \sin (\omega t) & \text { Equation } \mathbf{4} \\
a(t)=-\omega^{2} A \cos (\omega t) & \text { Equation 5 }
\end{array}
$$

## Procedure

## I. Pendulum Data and Analysis

1) Fasten the string to the metal ball and use the knob on the support bracket to hang it. Set the length of the pendulum to $\ell=80 \mathrm{~cm}$ (Figure 1).


Figure 1
2) Connect the motion detector to the Vernier interface. Adjust the height of the detector in such a way that it stays at the same level of the steel ball when it is in its equilibrium position (Photos 1 and 2). Keep the detector 80 cm from the pendulum.


Photo 1


Photo 2
3) Open the file "02 Pendulum" from the "Physics with Computers" file. Displace the ball 15 cm from its equilibrium position and release. Make sure the metal ball does not get too close to the detector. Click the Collect $D$ button. Your display should resemble Graph 1.
4) With the $X$ button measure the time interval between two consecutive crests on the Position-Time graph. Highlight this area with the $X$ button and record this time in the "Measured T" column. Print only the Position-Time graph, i.e. go to the File menu and choose "Print Graph").
5) Repeat Steps 3 and 4 until you have completely filled in the data table in Part I of your data sheet. You do not need to print out additional Position-Time graphs. Answer the questions at the end of Part I.

## II. Spring Data and Analysis

1) Hang the large spring to the support bracket. Hook the mass hanger from the lower end of the spring. Use a rubber band to secure the hanger to the spring. Place the motion detector on the floor directly below the mass hanger. Adjust the support until the bottom of the hanger is 70 cm above the detector (Figure 2 and Photo 3).


Figure 2


Photo 3
2) Place a 100 -gram slotted mass on the hanger and secure it with a rubber band (making a total mass of 0.150 kg ). Do not drop the hanger on the detector! Make sure the hanger is directly above the detector (Photos 4 and 5).


Photo 4


Photo 5
3) Open the file "15 Simple Harmonic Motion" from "Physics with Computers" file. Pull the mass hanger exactly 5 cm downward and release. Click the Collect $\triangleright$ button. Your display should resemble Graph 1.
4) Repeat Steps 3 and 4 until you have completely filled in the data table in Part II of your data sheet. Answer the questions at the end of Part II.

Each student is required to submit a completed data sheet in order to receive full credit. Your lab group needs to submit only one Position-Time graph. This graph is to be stapled to the data sheet of one of your lab partners - Each lab partner does not need to submit his/her own graph.


Graph 1


## Graph 2

## Data Sheet - Simple Harmonic Motion I

Name $\qquad$ Date $\qquad$
Partners' Names $\qquad$

## I. Pendulum Data and Analysis

| Type of Ball | Length [m] | Measured T [s] | Calculated T* [s] | \% discrepancy |
| :---: | :---: | :---: | :--- | :--- |
| Metal | 0.80 |  |  |  |
| Metal | 0.60 |  |  |  |
| Metal | 0.40 |  |  |  |
| Wood | 0.80 |  |  |  |
| Wood | 0.60 |  |  |  |
| Wood | 0.40 |  | *Use Equation 1 |  |

Answer the following questions.

1) Does the type of ball have any significant effect on the measured period?
2) Does the length of the pendulum affect the measured period? If so, explain how.

Use the $X$ button to answer Questions 3 through 10.
Select a point at either a crest ("hill') or a trough ("valley") along a Position-Time graph.
3) What is the velocity value on your Velocity-Time graph at these times?
4) Describe the motion of the mass at these times.

Select a point at either a crest ("hill') or a trough ("valley") along a Velocity-Time graph. 5) What is the position value on your Position-Time graph at these times?
6) What is the acceleration (=slope of the Velocity-Time graph) at these times?
7) Describe the motion of the mass at these times.

Find the points along your Velocity-Time graph where the acceleration is the greatest (i.e. largest positive number) and where the acceleration is the least (i.e. largest negative number.)
8) What is the position value on your Position-Time graph at these times?
9) What is the velocity value on your Velocity-Time graph at these times?
10) Describe the motion of the mass at these times.

## II. Spring Data and Analysis

| Total Mass $^{\dagger}[\mathrm{kg}]$ | Amplitude [m] | Frequency [Hz] | Measured T [s] |
| :---: | :--- | :--- | :--- |
| 0.150 |  |  |  |
| 0.200 |  |  |  |
| 0.250 |  |  |  |
| 0.300 |  |  |  |
| 0.350 |  |  |  |
| 0.400 |  |  |  |

${ }^{\dagger}$ Total mass here includes the mass of the hanger itself ( 50 grams).
Answer the following questions.

1) Does the mass affect the measured period? If so, explain how.
$2)$ Is the size of the amplitude related to the period of oscillation?
2) Use Equation 2 with your data for Total Mass $=0.400 \mathrm{~kg}$ to determine the spring constant $k$.
