

Work and Energy

Objectives

In this lab you will

- explore the relationship between the work done by an applied force and the area under the Force-Position graph.
- confirm that work is equivalent to a change in mechanical energy.
- learn how to use the Vernier Force Sensor.

Equipment

Vernier Force Sensor, Motion Detector, and LabPro™ system (includes computer and Logger Pro™), large spring, mass hanger, 150-gram slotted mass, rubber band, clamp, rod, and meterstick.

Theory

Suppose a constant external force F acts at an angle θ (to the direction of motion) on an object that moves a distance d . The work done by the force is given by

$$W = Fd \cos\theta \quad \text{Equation 1}$$

If F acts to raise a mass m from an initial height h_i to a final height h_f at constant speed, then $F = mg = \text{constant}$ and the work done is

$$W = mg(h_f - h_i) = \Delta U_{\text{grav}} \quad \text{Equation 2}$$

where $U_{\text{grav}} = mgh$ is defined as the gravitational potential energy. In this case, $\cos\theta = \cos 0^\circ = 1$ because F is parallel to the displacement.

Whenever the applied force is not constant, Equation 1 cannot be used to evaluate the work done. Hooke's Law is an example of such a variable force. Let F act against a Hooke's Law spring to displace it from position x_i to position x_f at constant speed. Then $F = kx$, where k is the spring constant. The work done against the spring is given by

$$W = \frac{1}{2}k(x_f^2 - x_i^2) = \Delta U_{\text{spring}} \quad \text{Equation 3}$$

where $U_{\text{spring}} = \frac{1}{2}kx^2$ is defined as the spring (elastic) potential energy. In this case, the work is identified as the area under Force-Position graph between x_i and x_f (see Figure 1). According to the language of calculus, this area is called the *integral* and the mathematical process of computing this area is *integration*.

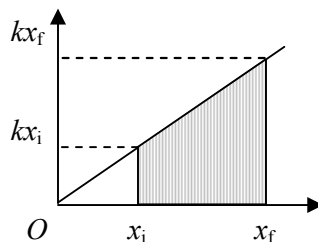


Figure 1

Data Collection and Analysis

Work Done Against Gravity

- 1) Connect the motion detector to the “DIG/SONIC1” port and the force sensor to “Channel 1” of the Lab Pro interface. Select ± 10 N on the force sensor. Open file “18a Work and Energy” from *Physics with Computers* file. Set the system to collect data for 5 sec. Click the ZERO button to insure that the force sensor reads 0 N when no force is being applied.
- 2) Fasten the slotted mass onto the hanger with the rubber band and then hang the assembly from the force sensor. Ask your partner to hold the meter stick vertically next to the mass hanger. Lay the motion detector on the floor. Position the bottom of the hanger at the 50-cm mark (see Figure 2). Please be extra careful not to drop the hanger on the detector!
- 3) Click the Collect \triangleright button. Wait one second after you hear the clicking sound to raise the hanger slowly and steadily away from the detector. The bottom of the hanger should now be in line with the top of the meter stick. Your computer should be displaying the associated Force-Time, Position-Time, and Force-Position graphs (see Graph 1).

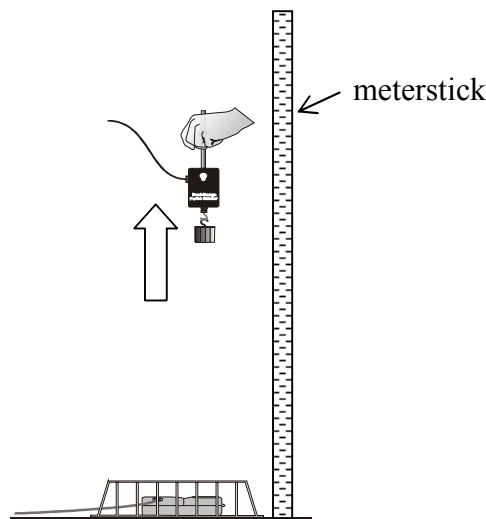
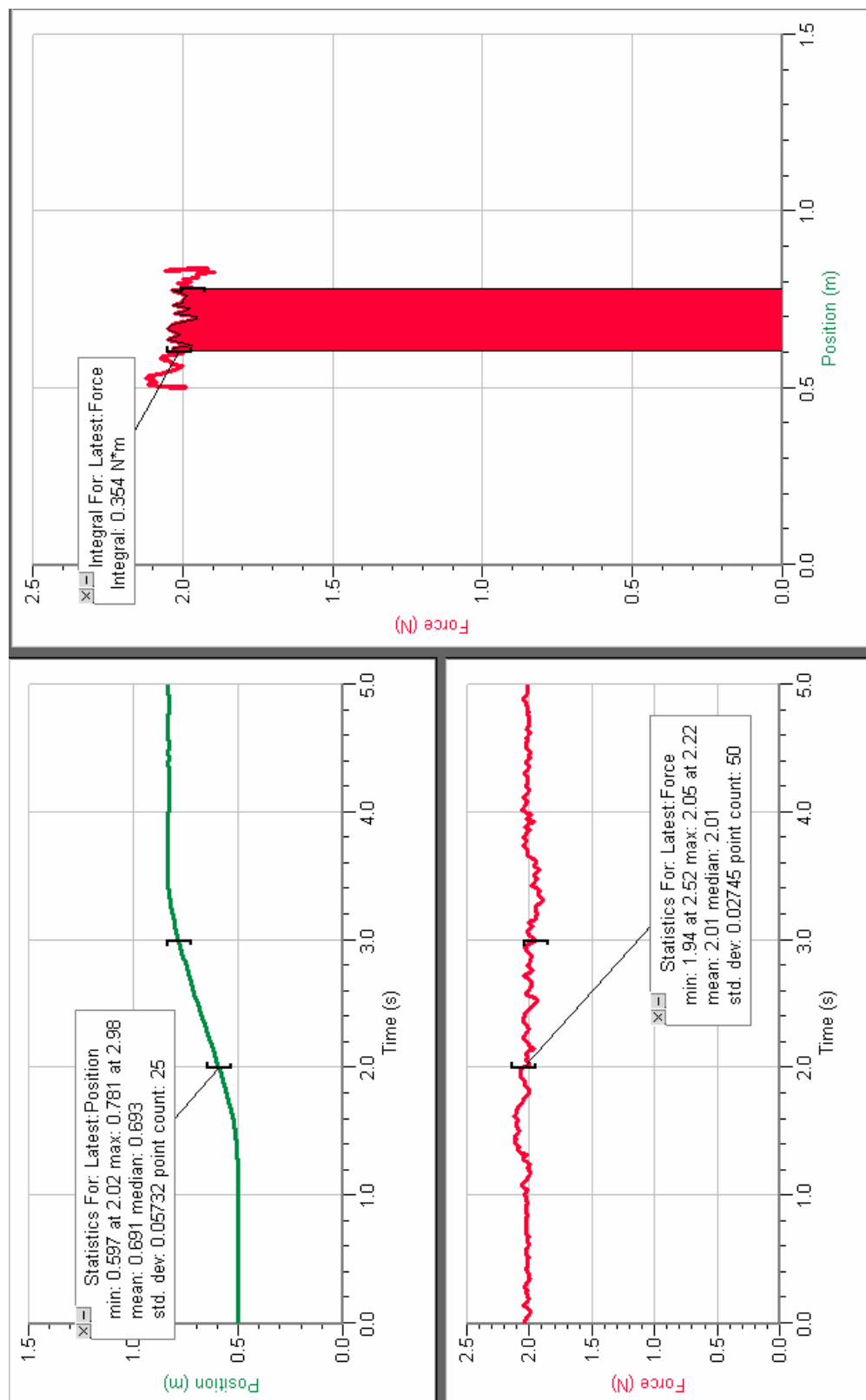


Figure 2

- 4) Click on the Position-Time and Force-Time graphs. Highlight the area between the time you started moving the hanger and the time you stopped. Select an initial position slightly after the hanger started moving. Select a final position slightly before the hanger stopped moving. This is to avoid including additional jitter in your data. Use the STAT button to find the average value of the force on the second graph for the same area as the previous graph and record this value on the data sheet as mg (see Graph 1).
- 5) Calculate (by hand) the change in the gravitational potential energy of the hanger at the initial and final positions that you have recorded (Equation 2). Record this value on your data sheet.



Graph 1

- 6) Highlight the area under your Force-Position graph between the initial and final positions that you selected in Step 4. Use the INTEGRATE button to find the area under the Force-Position graph between these two points. Record this area on your data sheet.
- 7) Now compare this area with the change in gravitational potential energy that you calculated in Step 5. Answer Questions 1-3 on data sheet.
- 8) Print these three graphs.

Work Done Against a Hooke's Law Spring

- 1) Open the file "18b Work and Energy."
- 2) Clamp the rod perpendicularly to the table. Clip one end of the spring to this rod and the other end to the force sensor. Place the force sensor on the table about 1 m away from the motion detector (see Photo 1 and Figure 3). Lay the meter stick between the sensor and the detector as shown.

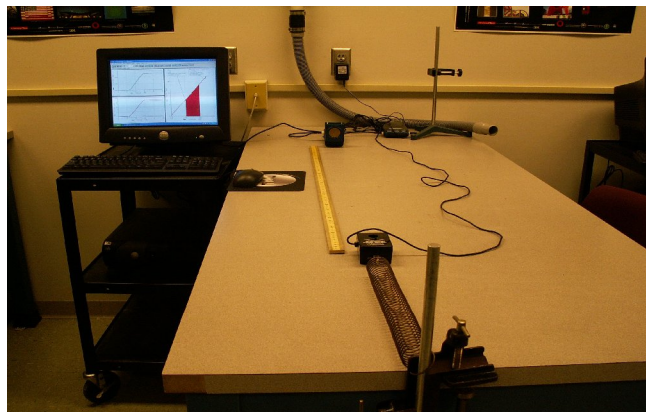


Photo 1

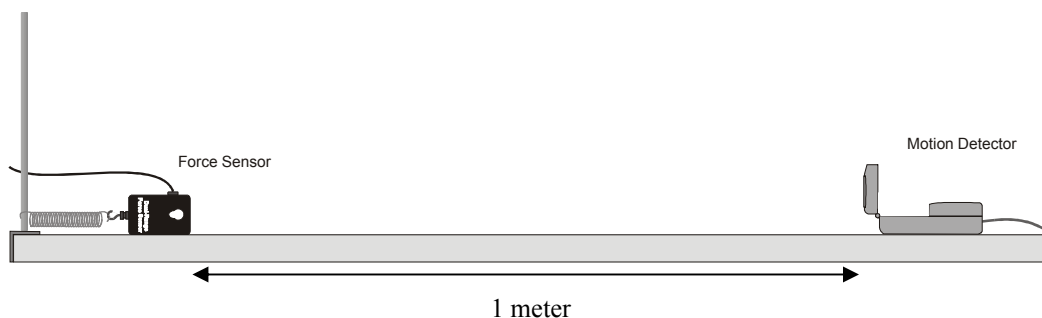
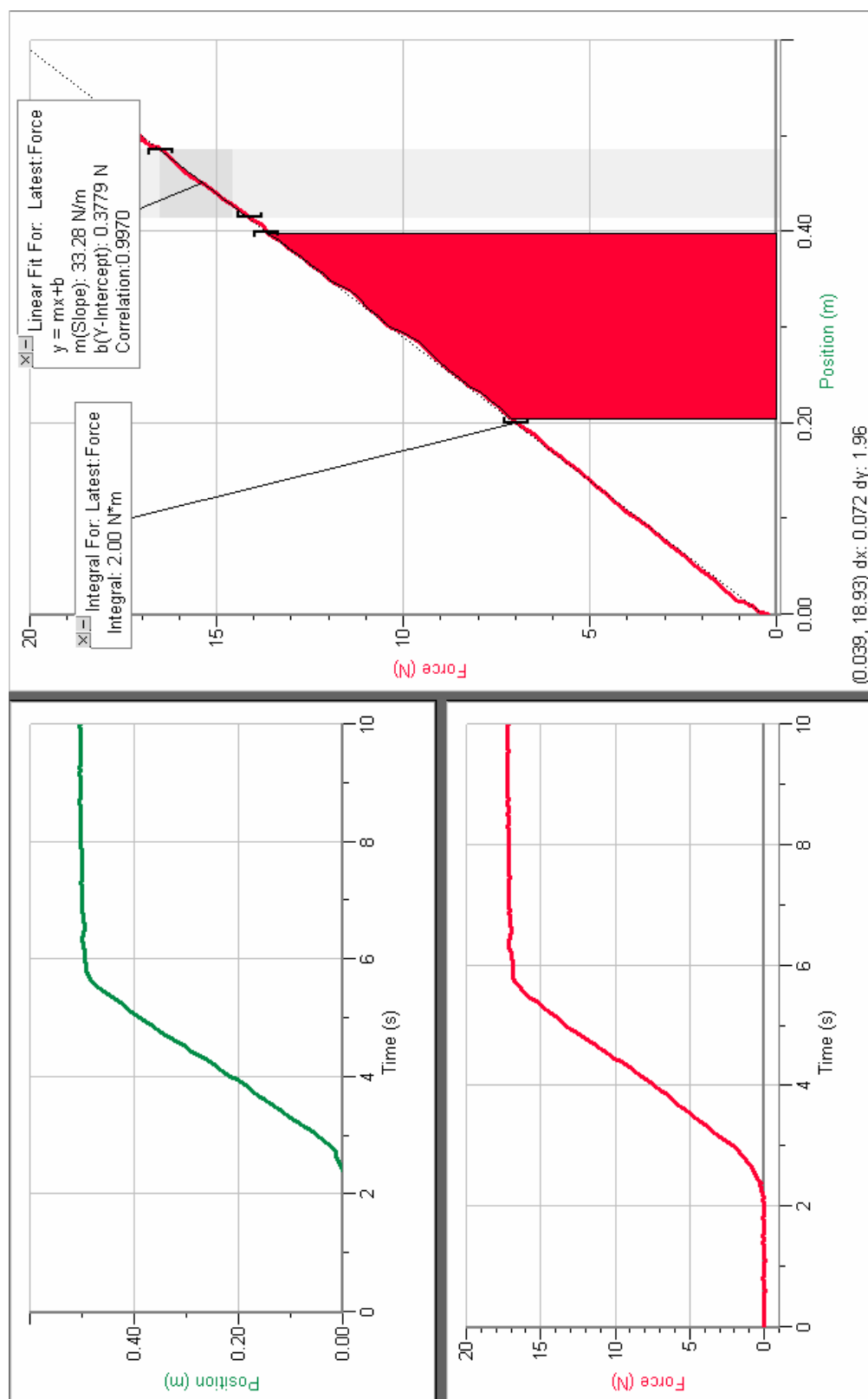


Figure 3

- 3) Keep the spring relaxed. Select ± 50 N on the force sensor. Close and re-open “Logger Pro 3” and “Use the Chosen Sensitivity” to switch over to the 50-N sensitivity.
- 4) Click the ZERO button to calibrate both the force sensor and motion detector. As a result, your Position-Time and Force-Time graphs should start at zero when you begin collecting data.
- 5) Now click Collect \triangleright and record data for 10 seconds. Wait one second and then move the force sensor smoothly toward the motion detector. When you have moved the sensor about 50 cm, stop – *Do not move the sensor until the clicking has ended.* Your display should closely resemble Graph 2. Note that the Force-Position graph is a straight line (Hooke’s law $F_s = kx$) starting at the origin.
- 6) Select a smooth portion of your Force-Position graph and use the LINEAR FIT button to find the slope, which corresponds to the spring constant k . Record the spring constant on your data sheet.
- 7) Highlight the area under your Force-Position graph between 0.00 and 0.20 m and then click the INTEGRATE button. Now highlight the area between 0.00 to 0.40 m and click INTEGRATE again. Record these two areas on the data sheet. Print these 3 graphs.
- 8) Use Equation 3 to calculate (by hand) the change in elastic potential energy for both the 0.20-m displacement and the 0.40-m displacement. Record these values on your data sheet and answer Questions 1-3.

Each student is required to submit a completed data sheet in order to receive full credit. Your lab group needs to submit only one set of graphs. These graphs are to be stapled to the data sheet of one of your lab partners – each lab partner does not need to submit his/her own set of graphs.



Graph 2

Data Sheet – Work and Energy

Name _____

Date _____

Partners' Names _____

Part I – Work Done Against Gravity

Initial height h_i [meter]	
Final height h_f [meter]	
Average force $F = mg$ [newtons]	
$\Delta U_{\text{grav}} = mg(h_f - h_i)$ [joule]	
Area under Force-Position graph, A	
% discrepancy = $\frac{ A - \Delta U_g }{A} \times 100$	

Answer the following questions.

- 1) What is the unit for the area under the Force-Position graph? Why?
- 2) Does your data confirm (i.e. is your percent discrepancy very low, say $< 2\%$?) that the work done against gravity is equivalent to the area under the Force-Position graph?
- 3) We have assumed that frictional losses are negligible in this experiment. How would your results change if frictional losses were significant?

Part II – Work Done Against a Hooke's Law Spring

Spring constant k [N/m]	
Area between 0.00 and 0.20 m, $A_{0.20}$	
$\Delta U_{0.20} = \frac{1}{2}k(0.20^2 - 0.00^2)$ [joule]	
% discrepancy = $\frac{ A_{0.20} - \Delta U_{0.20} }{A_{0.20}} \times 100$	
Area between 0.00 and 0.40 m, $A_{0.40}$	
$\Delta U_{0.40} = \frac{1}{2}k(0.40^2 - 0.00^2)$ [joule]	
% discrepancy = $\frac{ A_{0.40} - \Delta U_{0.40} }{A_{0.40}} \times 100$	

Answer the following questions.

1) Does your data confirm (i.e. are your percent discrepancies very low, say $< 2\%$?) that the work done against a Hooke's Law spring is equivalent to the area under the Force-Position graph?

2) What is name of the geometrical shape that bounds the areas under your Force-Position graph (see Figure 1)?

3) Derive Equation 3 algebraically using only geometrical area formulas. (See your textbook for additional help.)