

Centripetal Force

Objectives

In this lab you will

- test Newton's 2nd Law as it applies to uniform circular motion.
- determine the error in measuring period, radius, and mass and use these values to calculate the error in the centripetal force.
- confirm whether the calculated centripetal force agrees with its measured value using elementary error analysis.

Equipment

Centripetal force apparatus, a set of slotted masses, mass hanger, a mass scale, string, rubber bands, a ruler, and a stopwatch.

Theory

An object in *uniform circular motion* has constant speed while traveling in a circle. Such an object accelerates because its velocity vector is continuously changing its direction (see Figure 1). According to Newton's 2nd Law, a net external force is responsible for the acceleration:

$$F_c = ma_c = \frac{mv^2}{r} \quad \text{Equation 1}$$

F_c represents the net centripetal force, v is the speed, and r the radius of the circular path.

The term $a_c = v^2/r$ is the centripetal acceleration. The term "centripetal" simply means "directed toward the center" – it is not a description of a special category of force. For example, for a car traveling along a circular track, $F_c = f_s$, the static friction force that prevents the car from sliding off the track. For a ball attached to a string and rotated in a horizontal circle, $F_c = F_t$, the tension in the string.

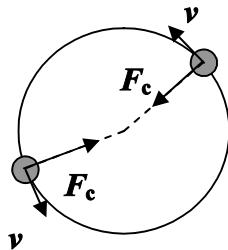


Figure 1 – Object of mass m in uniform circular motion (radius r)

Suppose an object makes N rotations in a time t . The *period* of rotation T is t / N . If the radius r is known, the speed v of the object may be calculated as

$$v = \frac{2\pi r}{T} \quad \text{Equation 2}$$

Substituting Equation 2 into Equation 1 yields an expression for F_c in terms of m , r , and T :

$$F_c = \frac{4\pi^2 mr}{T^2} \quad \text{Equation 3}$$

Error Analysis

As you learned in the Measurement and Errors lab, all experimental measurements are inherently limited and imprecise. An *error* is a numerical estimate of the size of these limitations and imprecisions. In this lab, you will be comparing two different experimental results for the centripetal force. One result is obtained from Equation 3. Because Equation 3 depends upon the measurements of m , r , and T and their associated errors δm , δr , and δT , our final answer F_c has an error δF_c given by*

$$\delta F_c = F_c \left[\left(\frac{\delta m}{m} \right) + \left(\frac{\delta r}{r} \right) + 2 \left(\frac{\delta T}{T} \right) \right] \quad \text{Equation 3}$$

The other result for the centripetal force comes from a direct measurement of the hanging weight $w = Mg$ of a mass M that provides an equivalent force. In this case, only the mass M must be measured. The error in the hanging weight δw is given by*

$$\delta w = g \delta M \quad \text{Equation 4}$$

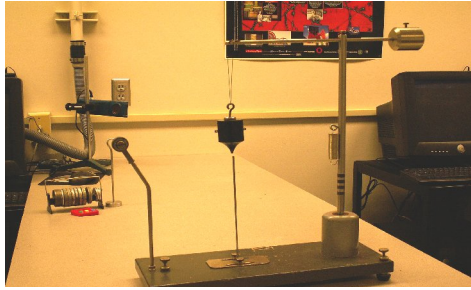
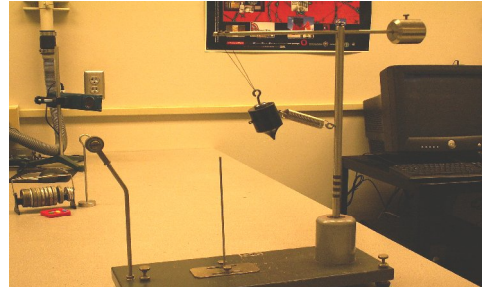
where $g = 9.80 \text{ m/s}^2$ and δM is the error in mass M .

* It is beyond the scope of this course to show you the details of how errors in independent variables (such as m , r , and T) propagate through a formula to generate an error in the dependent variable (such as F_c). The theory of error propagation requires a working knowledge of differential calculus and elementary statistics.

Procedure

Set-Up and Data Collection

- 1) Set up the apparatus as shown in Photo 1. Mass m is suspended from the horizontal bar that is tied to the vertical shaft. A counter weight attached to the opposite end of the bar.
- 2) Connect the spring to (suspended) mass m . Adjust the height of mass m such that when the attached spring is horizontal, mass m is close to the moveable marker (see Photo 2). This moveable marker indicates the position of mass m .
- 3) Use the adjusting screws to level the apparatus.
- 4) Measure mass m and record its value on your data sheet.

**Photo 1****Photo 2**

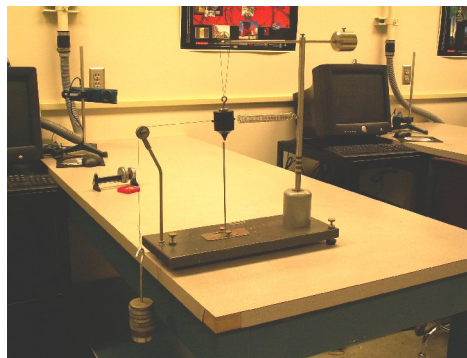
5) Attach the crossbar to the vertical shaft. Use the screw on the vertical shaft to hold the flat part of the crossbar. Use the counter weight for balancing the crossbar. Put the marker under mass m and adjust the length of string so the tip of mass m clears the marker (Photo 1). Attach the spring between mass m and the vertical shaft. This pulls the mass toward the shaft so that it no longer hangs vertically over the marker (Photo 2).

6) Twirl the shaft with your finger. Increase the speed until the tip of mass m returns on its own to its former position over the marker. Once this has occurred, maintain this speed. Use the stopwatch to time 20 complete rotations. Divide your time by 20 and record your result under period T in your data sheet. Repeat this procedure four more times and compute the average period.

7) Measure and record the radius of rotation r . Estimate the error in reading the ruler and record it as δr . Reading errors are typically $\frac{1}{2}$ of the smallest division.

8) Stop the rotation. Attach a piece of string between the mass hanger and mass m , then pass this string over the pulley (Photo 3). Add slotted masses to the hanger until mass m has been pulled to its former position over the marker. The total weight w of the hanger + slotted masses is now identical to the centripetal force. Record this hanging mass M (mass of hanger + slotted masses).

9) Complete side 1 of your data sheet. Turn to side 2 and perform a second trial with a different radius.

**Photo 3**

Each student is required to submit a completed data sheet in order to receive full credit.

Data Sheet – Centripetal Force

Name _____ Date _____

Partners' Names _____

Data Collection: Trial 1Mass $m =$ _____ kg Fractional error $\frac{\delta m}{m} = 0.05$

Period of rotation:

T_1 [sec]	T_2 [sec]	T_3 [sec]	T_4 [sec]	T_5 [sec]

Average period $T =$ _____ sec Fractional error $\frac{\delta T}{T} = 0.01$ Radius of rotation $r =$ _____ m Error $\delta r =$ _____ mFractional error $\frac{\delta r}{r} =$ _____Hanging mass $M =$ _____ kg Fractional error $\frac{\delta M}{M} = 0.05$ **Error Analysis: Trial 1**Centripetal force $F_c = \frac{4\pi^2 mr}{T^2} =$ _____ NError $\delta F_c = F_c \left[\left(\frac{\delta m}{m} \right) + \left(\frac{\delta r}{r} \right) + 2 \left(\frac{\delta T}{T} \right) \right] =$ _____ NWeight $w = Mg =$ _____ NError $\delta w = g \delta M =$ _____ NCompute the intervals $[F_c - \delta F_c, F_c + \delta F_c]$ and $[w - \delta w, w + \delta w]$.Do they overlap? If yes, then F_c and w are in agreement.

Data Collection: Trial 2Mass $m =$ _____ kgFractional error $\frac{\delta m}{m} = 0.05$

Period of rotation:

T_1 [sec]	T_2 [sec]	T_3 [sec]	T_4 [sec]	T_5 [sec]

Average period $T =$ _____ secFractional error $\frac{\delta T}{T} = 0.01$ Radius of rotation $r =$ _____ mError $\delta r =$ _____ mFractional error $\frac{\delta r}{r} =$ _____Hanging mass $M =$ _____ kgFractional error $\frac{\delta M}{M} = 0.05$ **Error Analysis: Trial 2**Centripetal force $F_c = \frac{4\pi^2 mr}{T^2} =$ _____ NError $\delta F_c = F_c \left[\left(\frac{\delta m}{m} \right) + \left(\frac{\delta r}{r} \right) + 2 \left(\frac{\delta T}{T} \right) \right] =$ _____ NWeight $w = Mg =$ _____ NError $\delta w = g\delta M =$ _____ NCompute the intervals $[F_c - \delta F_c, F_c + \delta F_c]$ and $[w - \delta w, w + \delta w]$.Do they overlap? If yes, then F_c and w are in agreement.