

## Measurement: The Basics

### I. Introduction

Physics is first and foremost an experimental science, meaning that its accumulated body of knowledge is due to the meticulous experiments performed by teams of researchers throughout the world. These experiments yield measurements that give physicists a peek into the inner workings of nature. The data is then analyzed in an attempt to uncover subtle relationships between various measured quantities such as length, time, mass, force, voltage, etc. Mathematics is used at every step of the experimental process, from the initial design phase to the final analysis of the data. You cannot begin to fully understand nor appreciate the central role of experimentation in physics unless you are introduced to the mathematics typically used in a physics laboratory. You also need to familiarize yourself with lab terminology and a few basic techniques before you perform your first physics experiment.

In this lab you will learn how to

- write measurements correctly, briefly, and clearly, using significant figures and error notation.
- determine the number of significant figures in any experimental result.
- distinguish between random and systematic errors and how they affect the accuracy and precision of your data.
- plot your data properly and draw the best-fit line.

### II. Significant Figures

Experiments are never perfect; there is no such thing as an exact measurement. In other words, measurements always have *uncertainties* that reflect the inherent limitations of the experiment. Because of these obvious though important facts, we must take extra care in the way we record our data. For example, suppose you measure the thickness of your textbook with an ordinary ruler and get 4.2 cm. Your result is said to be reliable to the nearest millimeter (0.1 cm); any digits smaller than 1 mm are uncertain simply because you were unable to measure them. It is inappropriate to record your measurement as 4.20 cm or 4.200 cm or 4.2000...cm (with an infinite number of trailing zeroes) because you did not measure those additional digits. In fact, you could write your measurement as 4.2????.. cm, where the question marks remind us that those digits are uncertain.

Now suppose you used a Vernier caliper instead of a ruler and measured the thickness to be 4.235 cm. This new measurement is more reliable since the Vernier caliper measures to the nearest 0.01 mm (0.001 cm) and has less uncertainty than your previous value. Nevertheless, there are still digits lower than 0.01 mm that have not been measured: 4.235????... cm. It should be abundantly clear at this point that there is no way to avoid the presence of uncertain digits in any measured number. We must constantly be mindful of this limitation whenever we record data directly from our measuring devices and also whenever we use our data in formulas. A quick and easy way to identify digits that are certain or uncertain is through significant figures.

A digit is a *significant figure* if it is

- (a) any nonzero number;
- (b) a trailing zero after the decimal point;
- (c) any zero between two significant digits.

Examples:

3.00	3 sig figs	1.0070	5 sig figs
800	1 sig fig	800.0	4 sig figs
$3.24 \times 10^{-5}$	3 sig figs	0.00324	3 sig figs

### III. Calculations with Significant Figures

Suppose we add 1.29 and 4.5. If these are “perfect” numbers, then their sum is obviously 5.79. But, if these numbers are measured, then we need to take into account their uncertainties in order to properly compute the sum:

$$\begin{array}{r} 1.29???... \\ + 4.5???? \dots \end{array}$$

But how is it possible to add certain digits to uncertain digits such as  $9 + ?$  in the third column? We need to use the following rule:

- **Whenever you add or subtract experimental numbers, the result should have the greatest uncertainty (i.e. fewest digits to the right of the decimal).**

In the above example, our final answer must be rounded to the nearest 0.1 since 4.5 has the greater uncertainty. Our answer is 5.8.

Other arithmetic operations also have special rules to keep the uncertainties in check:

- **Whenever you multiply or divide experimental numbers, the result should have the least number of significant figures.**
- **Whenever you raise an experimental number to a power or root, the result should have the same number of significant figures.**

Examples:

$$123,400 \div 5 = 20,000 \text{ (1 sig fig)}$$

$$\sqrt{144.0} = 12.00 \text{ (4 sig figs)}$$

$$2\pi(6.60 \times 10^{-15}) = 4.14 \times 10^{-14} \text{ (3 sig figs)}$$

Note in the last example that 2 and  $\pi$  are perfect numbers, which have no uncertainty! This is true for all integers, rational numbers, and special mathematical constants that appear in physics formulas ( $-5$ ,  $\frac{3}{4}$ ,  $e$ ,  $\sqrt{2}$ , etc.)

## IV. Errors, Precision, and Accuracy

The term *error* is a synonym for the uncertainty that is present in every measurement. Experimental errors are *not* mistakes – mistakes can be avoided, errors cannot. There are two principal types of errors:

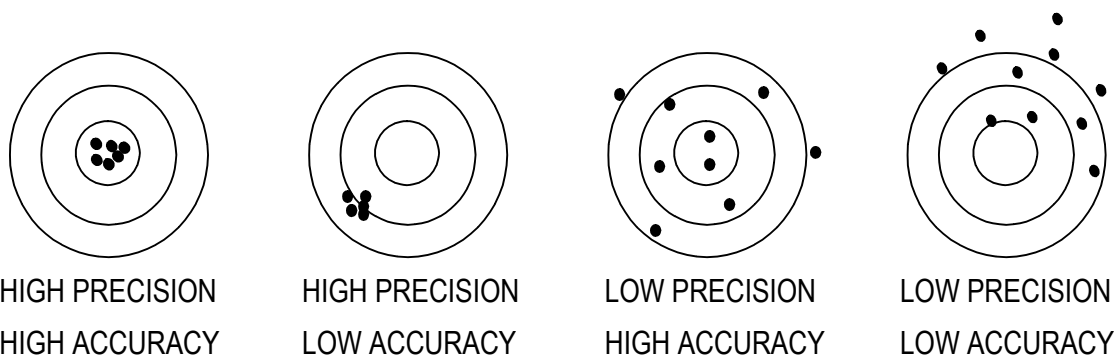
- Random errors cause random variations in repeated measurements. Each measurement is different by a different amount: some are bigger, some are smaller. Random errors are minimized by taking many measurements and computing the average. The average value is regarded as the value that best represents the data, because the fluctuations tend to cancel each other out when you sum the individual measurements.

- Systematic errors shift each measurement by the same amount in the same direction. Unlike random errors, we cannot minimize systematic errors by simply repeating the experiment many times. Instead we must make sure that our equipment is properly calibrated and that other experimental controls are in place before we begin collecting data.

There are two other terms that are closely associated with errors:

- The precision of a set of measurements is inversely related to the size of the random error. Data that is very precise “clusters” tightly together, indicating small random error.
- The accuracy of a set of measurements is inversely related to the size of the systematic error. Data that is very accurate is centered around an accepted standard value, indicating small systematic error.

A dartboard best illustrates the difference between precision and accuracy:



## V. Calculating Errors and Discrepancies

Whenever experimental results are presented in a lab report or in a scientific paper, all sources of error must be also reported and their effects explained in detail. Simply using the rules of significant figures is far too crude a method to express and contain errors in measurements. A better method involves explicitly recording the size of the error along with your measurement. The general format is

$$(\bar{x} \pm \delta x) \text{ units}$$

where  $\bar{x}$  is your best measurement (or the average value if you have more than one measurement) and  $\delta x$  is its *absolute error*.

Example:  $(9.8 \pm 0.2) \text{ m/s}^2$

In the above example,  $9.8 \text{ m/s}^2$  is the average value and  $0.2 \text{ m/s}^2$  is the error. The  $\pm$  indicates that the measurements range between  $9.6 \text{ m/s}^2$  and  $10.0 \text{ m/s}^2$ .

The *fractional* (or *relative error*) and the *percent error* are alternative ways of expressing the error:

$$\text{fractional error} = \frac{\delta x}{\bar{x}}$$

$$\text{percent error} = (\text{fractional error}) \times 100\%$$

Example: For the previous example, the fractional error is  $0.20/9.8 = 0.020$  and the percent error is 2.0%. You could express the result as  $9.8 \text{ m/s}^2 \pm 2.0\%$ .

It is often a matter of taste or expediency whether you choose to record your errors in absolute, fractional, or percent form.

When an experimental result is compared with a more reliable result (such as a result accepted by the international scientific community), the difference between the two is called the *discrepancy*, which can also be expressed in absolute, fractional, or percent form:

$$\text{absolute discrepancy} = |A - E|$$

$$\text{fractional discrepancy} = \frac{|A - E|}{A}$$

$$\text{percent discrepancy} = (\text{fractional disc}) \times 100\%$$

where  $A$  is the accepted result,  $E$  is the experimental result, and the bars  $| \cdot |$  indicate absolute value (discrepancies are always positive).

The following criterion is used to judge the accuracy of your results:

- **An experimental result agrees with an accepted value only if the discrepancy between the two values is numerically less than the error in the result. If this is case, we conclude that the result is accurate.**

Example:

Suppose you perform an experiment to measure the speed of sound in air (at STP). Your final result is  $342 \pm 1 \text{ m/s}$ . The accepted value is  $344 \text{ m/s}$ . The absolute discrepancy is  $|344 - 342| = 2 \text{ m/s}$  which is greater than the absolute error (1 m/s). Your result does not agree with the accepted value and, therefore, is inaccurate.

## VI. Graphing and Linear Interpolation

A graph is an accurate two-dimensional representation of experimental data. If you plot a graph by hand, use only high-quality graph paper (which you can purchase at the

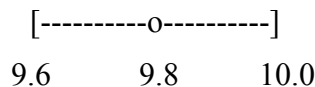
JAG Bookstore.) Cheaper graph paper often has slightly distorted gridlines that will affect the accuracy of your plot.

Every hand-drawn graph must conform to certain specifications:

- 1) Choose scales (tick marks) on your horizontal and vertical axes such that your data points are spread across the entire sheet.
- 2) Draw your axes with a sharp pencil. Label each axis and your tick marks. Record the units.
- 3) Use a sharp pencil to plot your points. Pens are acceptable but their marks cannot be erased.
- 4) Do not connect the dots with line segments! Instead, fit your data to a single straight line or a curve.
- 5) Include a brief title at the top that clearly describes what has been plotted.

In this introductory lab, you will only deal with data that follows a straight line. As you know, the equation of a line in the  $xy$  plane is given by  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  $y$ -intercept ( $y = b$  when  $x = 0$ ). To graph our experimental data we need to decide which quantity is the independent variable ( $x$ ) and which is the dependent variable ( $y$ ). If our measurements come with absolute errors, we need to include them as *error bars* in graph.

Think of an error bar as an interval along the  $x$  axis or the  $y$  axis. For example, in Part V we considered the measurement  $(9.8 \pm 0.2) \text{ m/s}^2$ . The error bar represents the interval  $9.6 \text{ m/s}^2 \leq x \leq 10.0 \text{ m/s}^2$ :



Once we have plotted our pairs of points ( $x, y$ ) along with their respective error bars, then we can draw the best-fit line through our data.

The *best-fit line* is that line that passes through the “middle” of the data. If you are plotting by hand, then it is wise to use a clear plastic ruler when you plot your best-line. Position the ruler so that there are approximately an equal number of points on each side of your straightedge. Of course, you would like all the points to line up along the straightedge. Since that rarely if ever occurs, you try to “split the difference” by finding a balance. When you have found the middle, scribe your line.

To determine the slope of the best-fit line, pick two points ( $x_1, y_1$ ) and ( $x_2, y_2$ ) along the line and use the standard formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

You are frequently asked to calculate the slope of the best-fit because it has a physical interpretation.

Note to Excel users: If you wish to plot the best-fit line and compute its slope, click on your graph. Now go to the Chart menu and select Add Trendline. Under Type, click

on Linear; under Options, check Display equation on chart. You should see your best-fit line and its equation in the form  $y = mx + b$ .

Example:

$x$ [s]	$1.0 \pm 0.4$	$2.0 \pm 0.3$	$3.0 \pm 0.3$
$y$ [m/s]	$3.1 \pm 0.5$	$5.5 \pm 0.9$	$6.4 \pm 0.5$

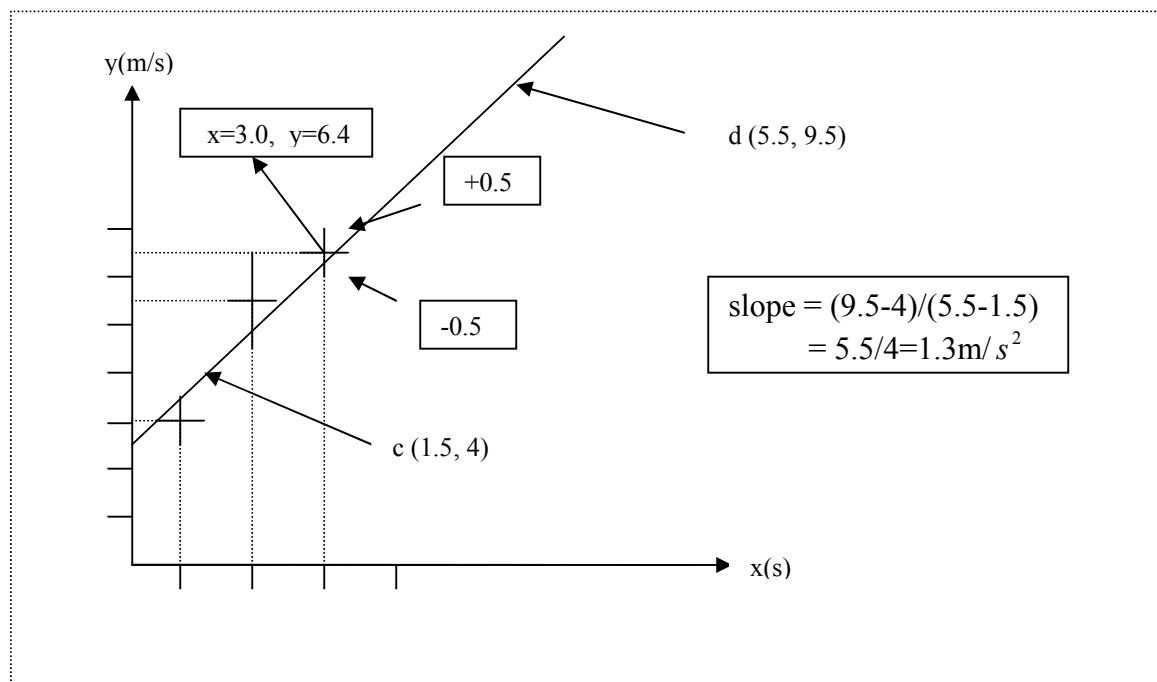


Figure 1. Graph of velocity vs. time. The slope represents acceleration.

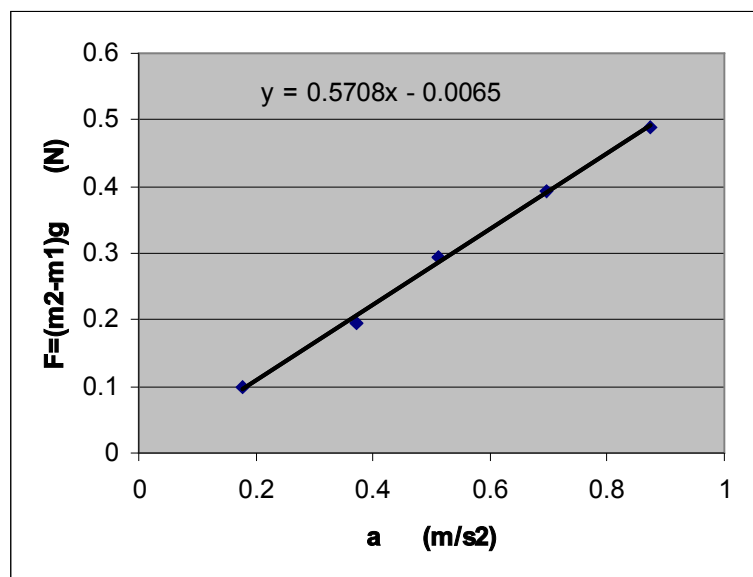


Figure 2. Graph of net force vs. acceleration.

## Measurement Worksheet

Write the number of significant figures in each number.

- |           |       |                       |       |
|-----------|-------|-----------------------|-------|
| 1) 0.150  | _____ | 4) 63.000             | _____ |
| 2) - 490  | _____ | 5) $0.75 \times 10^8$ | _____ |
| 3) 63,000 | _____ | 6) 00.001             | _____ |

Round the number 19.9605 to the specified number of significant figures.

- |               |       |               |       |
|---------------|-------|---------------|-------|
| 7) 4 sig figs | _____ | 9) 2 sig figs | _____ |
| 8) 3 sig figs | _____ | 10) 1 sig fig | _____ |

Perform the following calculations. Round your final answers according to the rules for significant figures (see Part III).

- |                                 |       |                                 |       |
|---------------------------------|-------|---------------------------------|-------|
| 11) $12.00 - 0.0462$            | _____ | 13) $\sqrt{(9.0 \times 10^6)}$  | _____ |
| 12) $\frac{(8.0)(1.50)}{4.222}$ | _____ | 14) $\frac{1}{2} (9.80)(0.5)^2$ | _____ |

Use a ruler to measure the length of each line segment. Estimate the absolute error in your measurement. Express your answer in the form  $(\bar{x} \pm \delta x)$  cm.

- |            |       |
|------------|-------|
| 15)  ----- | _____ |
| 16)  ----- | _____ |
| 17)  ----- | _____ |

18) Describe at least three experimental errors in measuring the length of the line segments.

19) Describe at least three ways to improve the accuracy of your measurements.

20) Explain how a wristwatch could be simultaneously very precise yet very inaccurate.  
(This is not a trick question!)

21) The following are measurements of the density of pure gold in  $\text{g/cm}^3$ :

19.0, 21.1, 20.8, 19.7, 19.9, 20.8, 20.6, 20.0, 20.7, 19.8

- (a) Calculate the average density. \_\_\_\_\_
- (b) Calculate the absolute error if each density is  $\pm 2\%$ . \_\_\_\_\_
- (c) Does your result in Part (a) agree with the accepted value of 19.3? Explain.

22)

$x$	1.5	2.3	3.7	4.2	5.3	6.9	7.8
$y$	6.6	9.4	14.8	16.4	20.8	26.3	29.9

- (a) Graph the above data either by hand or using Excel.  
Remember the graph specifications listed in Part VI?
- (b) Draw error bars at each point. (Assume  $\pm 10\%$  for each point.)
- (c) Draw the best-fit line and determine its slope and  $y$ -intercept.

slope  $m =$  \_\_\_\_\_

$y$ -intercept  $b =$  \_\_\_\_\_